

thm_2EquantHeuristics_2EGUESS_EXISTS_THM
(TMG-
JABCsf5kVYynSF1r1c89WfDX1WzYHyEZ)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (\text{c_2Emin_2E_40 } A \ 27a))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a})$

Definition 6 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t)).$

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q \ Q)$ of type $\iota.$

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2E_2F } 2))$

Definition 9 We define `c_2EquantHeuristics_2EGUESS_FORALL_GAP` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}).$

Definition 10 We define `c_2EquantHeuristics_2EGUESS_EXISTS_GAP` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A. 27b) (\lambda V2v \in A. 27b. ($

Definition 11 We define `c_2EquantHeuristics_2EGUESS_FORALL_POINT` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A. 27a) (\lambda V2fv \in A. 27a. ($

Definition 12 We define `c_2EquantHeuristics_2EGUESS_EXISTS_POINT` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A. 27a) (\lambda V2fv \in A. 27a. ($

Definition 13 We define `c_2EquantHeuristics_2EGUESS_FORALL` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}).$

Definition 14 We define `c_2EquantHeuristics_2EGUESS_EXISTS` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27a} \dots$

Definition 15 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 16 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}). ((p (ap (c_2Ebool_2E_3F A_{.27a}) V0f)) \Leftrightarrow (\exists V1x \in A_{.27a}. (p (ap V0f V1x))))) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (\neg (p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \tag{5}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t))))) \tag{7}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \tag{8}$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \tag{9}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p\ (ap\ (\\
& \quad ap\ (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ V1P))) \Leftrightarrow \\
& \quad (\forall V2v \in A_27b. ((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A_27a. (p\ (\\
& \quad ap\ V1P\ (ap\ V0i\ V3fv)))))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_27a\ A_27b)\ V0i)\ V1P))) \Leftrightarrow (\forall V4v \in A_27b. ((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow \\
& \quad (\exists V5fv \in A_27a. (\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv)))))) \wedge ((\forall V6i \in \\
& \quad (A_27b^{A_27a}). (\forall V7P \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_27a\ A_27b)\ V6i)\ V7P))) \Leftrightarrow (\forall V8fv \in A_27a. (p\ (ap\ V7P\ (ap\ V6i\ V8fv)))))) \wedge \\
& \quad ((\forall V9i \in (A_27b^{A_27a}). (\forall V10P \in (2^{A_27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27a\ A_27b) \\
& \quad V9i)\ V10P))) \Leftrightarrow (\forall V11fv \in A_27a. (\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))) \wedge \\
& \quad ((\forall V12i \in (A_27b^{A_27a}). (\forall V13P \in (2^{A_27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b) \\
& \quad V12i)\ V13P))) \Leftrightarrow (\forall V14v \in A_27b. ((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in \\
& \quad A_27a. (V14v = (ap\ V12i\ V15fv)))))) \wedge (\forall V16i \in (A_27b^{A_27a}). \\
& \quad (\forall V17P \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27a\ A_27b)\ V16i)\ V17P))) \Leftrightarrow (\forall V18v \in A_27b. ((\neg(p\ (ap\ V17P\ V18v))) \Rightarrow \\
& \quad (\exists V19fv \in A_27a. (V18v = (ap\ V16i\ V19fv))))))))) \\
& \hspace{10em} (10)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (13)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (14)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False))) \quad (15)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))) \quad (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee \neg(p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge ((\\
& \neg(p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow \neg(p \vee 1q)) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge (\neg(p \vee 1q) \vee \neg(p \vee 0p))))))
\end{aligned} \tag{19}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p))) \tag{20}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow \neg(p \vee 1q))) \tag{21}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). ((p \ (ap \ (ap \\
& (c_2EquantHeuristics_2EGUESS_EXISTS \ A_27a \ A_27b) \ V0i) \ V1P)) \Rightarrow \\
& ((p \ (ap \ (c_2Ebool_2E_3F \ A_27b) \ V1P)) \Leftrightarrow (\exists V2fv \in A_27a. (p \ (\\
& ap \ V1P \ (ap \ V0i \ V2fv))))))
\end{aligned}$$