

# thm\_2EquantHeuristics\_2EGUESS\_FORALL\_POINT\_THM (TMbj6FfSiESQUVkgGc29gAtJFbNS7CSbnyxY)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p (ap P x))$ )  
of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P) V0P))$

**Definition 6** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 8** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

**Definition 9** We define `c_2EquantHeuristics_2EGUESS_FORALL_GAP` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27a)$

**Definition 10** We define `c_2EquantHeuristics_2EGUESS_EXISTS_GAP` to be  
 $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27a^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2v \in A_27b.(ap V1P v)))$

**Definition 11** We define `c_2EquantHeuristics_2EGUESS_FORALL_POINT` to  
be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27a^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27a) (\lambda V2fv \in A_27a.(ap V1P fv)))$

**Definition 12** We define `c_2EquantHeuristics_2EGUESS_EXISTS_POINT` to be  
 $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27a^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27a) (\lambda V2fv \in A_27a.(ap V1P fv)))$

**Definition 13** We define `c_2EquantHeuristics_2EGUESS_FORALL` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27a)$

**Definition 14** We define `c_2EquantHeuristics_2EGUESS_EXISTS` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27a)$

**Definition 15** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).((p (ap (c\_2Ebool\_2E\_21 A\_27a) V0f)) \Leftrightarrow (\forall V1x \in A\_27a.(p (ap V0f V1x)))))) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{4}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False) \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p (ap V0P V2x)))))) \tag{8}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \tag{9}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \tag{10}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0i \in (A\_27b^{A\_27a}). (\forall V1P \in (2^{A\_27b}). (((p\ (ap\ ( \\
& \quad ap\ (c\_2EquantHeuristics\_2EGUESS\_EXISTS\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Leftrightarrow \\
& \quad (\forall V2v \in A\_27b. ((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A\_27a. (p\ ( \\
& \quad ap\ V1P\ (ap\ V0i\ V3fv)))))) \wedge ((p\ (ap\ (ap\ (c\_2EquantHeuristics\_2EGUESS\_FORALL \\
& \quad A\_27a\ A\_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V4v \in A\_27b. ((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow \\
& \quad (\exists V5fv \in A\_27a. (\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv)))))) \wedge ((\forall V6i \in \\
& \quad (A\_27b^{A\_27a}). (\forall V7P \in (2^{A\_27b}). ((p\ (ap\ (ap\ (c\_2EquantHeuristics\_2EGUESS\_EXISTS\_POINT \\
& \quad A\_27a\ A\_27b)\ V6i)\ V7P)) \Leftrightarrow (\forall V8fv \in A\_27a. (p\ (ap\ V7P\ (ap\ V6i\ V8fv)))))) \wedge \\
& \quad ((\forall V9i \in (A\_27b^{A\_27a}). (\forall V10P \in (2^{A\_27b}). ((p\ (ap \\
& \quad (ap\ (c\_2EquantHeuristics\_2EGUESS\_FORALL\_POINT\ A\_27a\ A\_27b) \\
& \quad V9i)\ V10P)) \Leftrightarrow (\forall V11fv \in A\_27a. (\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))) \wedge \\
& \quad ((\forall V12i \in (A\_27b^{A\_27a}). (\forall V13P \in (2^{A\_27b}). ((p\ (ap \\
& \quad (ap\ (c\_2EquantHeuristics\_2EGUESS\_EXISTS\_GAP\ A\_27a\ A\_27b) \\
& \quad V12i)\ V13P)) \Leftrightarrow (\forall V14v \in A\_27b. ((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in \\
& \quad A\_27a. (V14v = (ap\ V12i\ V15fv)))))) \wedge (\forall V16i \in (A\_27b^{A\_27a}). \\
& \quad (\forall V17P \in (2^{A\_27b}). ((p\ (ap\ (ap\ (c\_2EquantHeuristics\_2EGUESS\_FORALL\_GAP \\
& \quad A\_27a\ A\_27b)\ V16i)\ V17P)) \Leftrightarrow (\forall V18v \in A\_27b. ((\neg(p\ (ap\ V17P\ V18v))) \Rightarrow \\
& \quad (\exists V19fv \in A\_27a. (V18v = (ap\ V16i\ V19fv))))))))) \\
& \hspace{10em} (11)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (13)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (14)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \quad (15)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q)))) \quad (16)$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0i \in (A\_27b^{A\_27a}). (\forall V1P \in (2^{A\_27b}). ((p\ (ap\ (ap \\
& \quad (c\_2EquantHeuristics\_2EGUESS\_FORALL\_POINT\ A\_27a\ A\_27b)\ V0i) \\
& \quad V1P)) \Rightarrow ((p\ (ap\ (c\_2Ebool\_2E\_21\ A\_27b)\ V1P)) \Leftrightarrow False)))
\end{aligned}$$