

thm_2EquantHeuristics_2EGUESS_REWRITES (TMZN2JuepNCo6jdGK5kfD3Mm1dtMQzMsUTJ)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))$

Definition 4 We define `c_2EquantHeuristics_2EGUESS_FORALL` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b)$

Definition 5 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$
of type $\iota \Rightarrow \iota$.

Definition 6 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A_27a))$

Definition 7 We define `c_2EquantHeuristics_2EGUESS_EXISTS` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b)$

Definition 8 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$
of type ι .

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

Definition 10 We define `c_2EquantHeuristics_2EGUESS_EXISTS_POINT` to be
 $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27a) (\lambda V2fv \in A_27a. V2fv))$

Definition 11 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 12 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2E_2F } V0t))$

Definition 13 We define `c_2EquantHeuristics_2EGUESS_FORALL_POINT` to
be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27a) (\lambda V2fv \in A_27a. V2fv))$

Definition 14 We define `c_2EquantHeuristics_2EGUESS_EXISTS_GAP` to be
 $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27b) (\lambda V2v \in A_27b. (V2v \wedge P v)))$

Definition 15 We define `c_2EquantHeuristics_2EGUESS_FORALL_GAP` to be
 $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27b) (\lambda V2v \in A_27b. (V2v \wedge P v)))$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A-27a}). (\forall V1P \in (2^{A-27b}). (((p\ (ap\ (ap \\
& (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ V1P)) \Leftrightarrow \\
& \quad (\forall V2v \in A_27b. ((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A_27a. (p\ (\\
ap\ V1P\ (ap\ V0i\ V3fv)))))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_27a\ A_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V4v \in A_27b. ((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow \\
& \quad (\exists V5fv \in A_27a. (\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv)))))))))) \\
& \hspace{15em} (1)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A-27a}). (\forall V1P \in (2^{A-27b}). (((p\ (ap\ (\\
ap\ (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ V1P)) \Leftrightarrow \\
& \quad (\forall V2v \in A_27b. ((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A_27a. (p\ (\\
ap\ V1P\ (ap\ V0i\ V3fv)))))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_27a\ A_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V4v \in A_27b. ((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow \\
& \quad (\exists V5fv \in A_27a. (\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv)))))))) \wedge ((\forall V6i \in \\
(A_27b^{A-27a}). (\forall V7P \in (2^{A-27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_27a\ A_27b)\ V6i)\ V7P)) \Leftrightarrow (\forall V8fv \in A_27a. (p\ (ap\ V7P\ (ap\ V6i\ V8fv)))))) \wedge \\
& \quad ((\forall V9i \in (A_27b^{A-27a}). (\forall V10P \in (2^{A-27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27a\ A_27b)\ \\
V9i)\ V10P)) \Leftrightarrow (\forall V11fv \in A_27a. (\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))) \wedge \\
& \quad ((\forall V12i \in (A_27b^{A-27a}). (\forall V13P \in (2^{A-27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b)\ \\
V12i)\ V13P)) \Leftrightarrow (\forall V14v \in A_27b. ((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in \\
& \quad A_27a. (V14v = (ap\ V12i\ V15fv)))))) \wedge (\forall V16i \in (A_27b^{A-27a}). \\
(\forall V17P \in (2^{A-27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27a\ A_27b)\ V16i)\ V17P)) \Leftrightarrow (\forall V18v \in A_27b. ((\neg(p\ (ap\ V17P\ V18v))) \Rightarrow \\
& \quad (\exists V19fv \in A_27a. (V18v = (ap\ V16i\ V19fv))))))))))
\end{aligned}$$