thm_2EquantHeuristics_2EGUESS__RULES__BOOL (TMdGTbunDgQdfb-MzB55QJ65jo9eTEN2WiXx)

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Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (1)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 9 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 10 *We define* c_2EquantHeuristics_2EGUESS__FORALL__GAP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27b^{A\_27a}).\lambda V1P \in (2^{A\_27b}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27b)\ (\lambda V2v \in A\_27b.(a$

Definition 11 *We define* c_2EquantHeuristics_2EGUESS__EXISTS__GAP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27b^{A\_27a}).\lambda V1P \in (2^{A\_27b}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27b)\ (\lambda V2v \in A\_27b.(a$

Definition 12 *We define* c_2EquantHeuristics_2EGUESS__FORALL__POINT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27b^{A\_27a}).\lambda V1P \in (2^{A\_27b}).(ap$ (c_2Ebool_2E_21 $A\_27a)$ $(\lambda V2fv \in A\_27$

Definition 13 *We define* c_2EquantHeuristics_2EGUESS__EXISTS__POINT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27b^{A\_27a}).\lambda V1P \in (2^{A\_27b}).(ap$ (c_2Ebool_2E_21 $A\_27a)$ $(\lambda V2fv \in A\_27a.$

Definition 14 *We define* c_2EquantHeuristics_2EGUESS__FORALL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27$

Definition 15 *We define* c_2EquantHeuristics_2EGUESS__EXISTS *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27$

Definition 16 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$True \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{5}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{7}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27b}).((((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V2v \in A\_27b.((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A\_27a.(p\ (ap\ V1P\ (ap\ V0i\ V3fv)))))))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V4v \in A\_27b.((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow (\exists V5fv \in A\_27a.(\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv))))))))) \wedge ((\forall V6i \in (A\_27b^{A\_27a}).(\forall V7P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_POINT}\ A\_27a\ A\_27b)\ V6i)\ V7P)) \Leftrightarrow (\forall V8fv \in A\_27a.(p\ (ap\ V7P\ (ap\ V6i\ V8fv))))))) \wedge ((\forall V9i \in (A\_27b^{A\_27a}).(\forall V10P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_POINT}\ A\_27a\ A\_27b)\ V9i)\ V10P)) \Leftrightarrow (\forall V11fv \in A\_27a.(\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))))) \wedge ((\forall V12i \in (A\_27b^{A\_27a}).(\forall V13P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_GAP}\ A\_27a\ A\_27b)\ V12i)\ V13P)) \Leftrightarrow (\forall V14v \in A\_27b.((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in A\_27a.(V14v = (ap\ V12i\ V15fv)))))))) \wedge (\forall V16i \in (A\_27b^{A\_27a}).(\forall V17P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_GAP}\ A\_27a\ A\_27b)\ V16i)\ V17P)) \Leftrightarrow (\forall V18v \in A\_27b.((\neg(p\ (ap\ V17P\ V18v))) \Rightarrow (\exists V19fv \in A\_27a.(V18v = (ap\ V16i\ V19fv))))))))))))) \tag{10}$$

Theorem 1

$$((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_POINT}\ ty\_2Eone\_2Eone\ 2)\ (\lambda V0ARB \in ty\_2Eone\_2Eone.\mathsf{c\_2Ebool\_2ET}))\ (\lambda V1x \in 2. V1x))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_POINT}\ ty\_2Eone\_2Eone\ 2)\ (\lambda V2ARB \in ty\_2Eone\_2Eone.\mathsf{c\_2Ebool\_2EF}))\ (\lambda V3x \in 2.V3x))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_GAP}\ ty\_2Eone\_2Eone\ 2)\ (\lambda V4ARB \in ty\_2Eone\_2Eone.\mathsf{c\_2Ebool\_2ET}))\ (\lambda V5x \in 2.V5x))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_GAP}\ ty\_2Eone\_2Eone\ 2)\ (\lambda V6ARB \in ty\_2Eone\_2Eone.\mathsf{c\_2Ebool\_2EF}))\ (\lambda V7x \in 2.V7x))))))$$