

thm_2EquantHeuristics_2EGUESS_RULES_CONJ
(TM-
ZoMj5ZoUACMippf5fTFoM1DkxD1GA4krh)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 10 We define $c_2EquantHeuristics_2EGUESS_FORALL_GAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2v \in A_27b.(a$

Definition 11 We define $c_2EquantHeuristics_2EGUESS_FORALL_POINT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27a) (\lambda V2fv \in A_27b.(a$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 12 We define $c_2EquantHeuristics_2EGUESS_FORALL$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2fv \in A_27b.(a$

Definition 13 We define `c_2EquantHeuristics_2EGUESS_EXISTS_GAP` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0i \in (A.27b^{A.27a}).\lambda V1P \in (2^{A.27b}).(ap (c_2Ebool_2E_21 A.27b) (\lambda V2v \in A.27b.(a$

Definition 14 We define `c_2EquantHeuristics_2EGUESS_EXISTS` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0i \in (A.27b^{A.27a}).\lambda V1P \in (2^{A.27b}).(ap (c_2Ebool_2E_21 A.27b) (\lambda V2v \in A.27b.(a$

Definition 15 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t3 \in 2.((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))))))$

Definition 16 We define `c_2EquantHeuristics_2EGUESS_EXISTS_POINT` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0i \in (A.27b^{A.27a}).\lambda V1P \in (2^{A.27b}).(ap (c_2Ebool_2E_21 A.27a) (\lambda V2fv \in A.27a.(a$

Definition 17 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t3 \in 2.((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))))))$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (2)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))))) \quad (3)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3)))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (4)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))))) \quad (5)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c.nonempty A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}).(\forall V2x \in A.27c.((ap (ap (ap (c_2Ecombin_2Eo A.27c A.27b A.27a) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (6)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0i \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27b}). ((p\ (ap\ (ap \\
& \quad (c.2EquantHeuristics.2EGUESS_EXISTS\ A.27a\ A.27b)\ V0i)\ (\lambda V2x \in \\
& \quad A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V2x)))) \Leftrightarrow (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_FORALL \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V3x \in A.27b.(ap\ V1P\ V3x)))) \wedge ((p\ (ap\ (ap \\
& \quad (c.2EquantHeuristics.2EGUESS_FORALL\ A.27a\ A.27b)\ V0i)\ (\lambda V4x \in \\
& \quad A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V4x)))) \Leftrightarrow (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_EXISTS \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V5x \in A.27b.(ap\ V1P\ V5x)))) \wedge ((p\ (ap\ (ap \\
& \quad (c.2EquantHeuristics.2EGUESS_EXISTS_GAP\ A.27a\ A.27b)\ V0i) \\
& \quad (\lambda V6x \in A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V6x)))) \Leftrightarrow (p\ (ap\ (ap \\
& \quad (c.2EquantHeuristics.2EGUESS_FORALL_GAP\ A.27a\ A.27b)\ V0i) \\
& \quad (\lambda V7x \in A.27b.(ap\ V1P\ V7x)))) \wedge ((p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_FORALL_GAP \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V8x \in A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V8x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_EXISTS_GAP\ A.27a\ A.27b) \\
& \quad V0i)\ (\lambda V9x \in A.27b.(ap\ V1P\ V9x)))) \wedge ((p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_EXISTS_POIN \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V10x \in A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V10x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_FORALL_POINT\ A.27a \\
& \quad A.27b)\ V0i)\ (\lambda V11x \in A.27b.(ap\ V1P\ V11x)))) \wedge ((p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_FORALL_ \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V12x \in A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V12x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_EXISTS_POINT\ A.27a \\
& \quad A.27b)\ V0i)\ (\lambda V13x \in A.27b.(ap\ V1P\ V13x)))))))))
\end{aligned}$$

(7)

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\
& \quad \forall V0i \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27b}). (\forall V2Q \in \\
& \quad (2^{A.27b}). (\forall V3iK \in A.27b. (\forall V4q \in 2. (\forall V5p \in \\
& \quad 2. (((p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V6x \in A.27b.(ap V1P V6x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V7x \in A.27b.(ap (ap c.2Ebool.2E.2F.5C (ap \\
V1P V7x)) (ap V2Q V7x)))))) \wedge (((p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V8x \in A.27b.(ap V2Q V8x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V9x \in A.27b.(ap (ap c.2Ebool.2E.2F.5C (ap \\
V1P V9x)) (ap V2Q V9x)))))) \wedge (((p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V10x \in A.27b.(ap V1P V10x)))) \wedge (p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V11x \in \\
A.27b.(ap V2Q V11x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V12x \in A.27b.(ap (ap c.2Ebool.2E.2F.5C (\\
ap V1P V12x)) (ap V2Q V12x)))))) \wedge (((p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V13x \in A.27b.(ap V1P V13x)))) \wedge (p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V14x \in A.27b.(ap V2Q V14x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_FORALL_POINT \\
A.27a A.27b) V0i) (\lambda V15x \in A.27b.(ap (ap c.2Ebool.2E.2F.5C (\\
ap V1P V15x)) (ap V2Q V15x)))))) \wedge (((p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS \\
ty.2Eone.2Eone A.27b) (\lambda V16xxx \in ty.2Eone.2Eone.V3iK)) (\lambda V17x \in \\
A.27b.(ap V1P V17x)))) \wedge (p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS \\
ty.2Eone.2Eone A.27b) (\lambda V18xxx \in ty.2Eone.2Eone.V3iK)) (\lambda V19x \in \\
A.27b.(ap V2Q V19x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS \\
ty.2Eone.2Eone A.27b) (\lambda V20xxx \in ty.2Eone.2Eone.V3iK)) (\lambda V21x \in \\
A.27b.(ap (ap c.2Ebool.2E.2F.5C (ap V1P V21x)) (ap V2Q V21x)))))) \wedge \\
(((p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS A.27a A.27b) \\
V0i) (\lambda V22x \in A.27b.(ap V1P V22x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS \\
A.27a A.27b) V0i) (\lambda V23x \in A.27b.(ap (ap c.2Ebool.2E.2F.5C (\\
ap V1P V23x)) V4q)))))) \wedge (((p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS \\
A.27a A.27b) V0i) (\lambda V24x \in A.27b.(ap V2Q V24x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS \\
A.27a A.27b) V0i) (\lambda V25x \in \\
A.27b.(ap (ap c.2Ebool.2E.2F.5C V5p) (ap V2Q V25x)))))) \wedge (((p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS_POINT \\
A.27a A.27b) \\
V0i) (\lambda V26x \in A.27b.(ap V1P V26x)))) \wedge (p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS_POINT \\
A.27a A.27b) V0i) (\lambda V27x \in A.27b.(ap V2Q V27x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS_POINT \\
A.27a A.27b) V0i) (\lambda V28x \in A.27b.(ap (ap c.2Ebool.2E.2F.5C (ap V1P V28x)) (ap V2Q \\
V28x)))))) \wedge (((p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS_GAP \\
A.27a A.27b) V0i) (\lambda V29x \in A.27b.(ap V1P V29x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS_GAP \\
A.27a A.27b) V0i) (\lambda V30x \in A.27b.(ap (ap c.2Ebool.2E.2F.5C (ap V1P V30x)) (ap V2Q \\
V30x)))))) \wedge (((p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS_GAP \\
A.27a A.27b) V0i) (\lambda V31x \in A.27b.(ap V2Q V31x)))) \Rightarrow (p (ap (ap (c.2EquantHeuristics.2EGUESS_EXISTS_GAP \\
A.27a A.27b) V0i) (\lambda V32x \in A.27b.(ap (ap c.2Ebool.2E.2F.5C (ap V1P V32x)) (ap V2Q \\
V32x)))))))))
\end{aligned}$$