

# thm\_2EquantHeuristics\_2EGUESS\_\_RULES\_\_DISJ (TMYGZD8Q2aipcmqj6329mRVeSbZi6Hig2jA)

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Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (1)$$

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A . \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 9** We define  $c\_2EquantHeuristics\_2EGUESS\_FORALL\_GAP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0i \in (A\_27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (ap (c\_2Ebool\_2E\_21 A\_27b) (\lambda V2v \in A\_27b. (a$

**Definition 10** We define  $c\_2EquantHeuristics\_2EGUESS\_EXISTS\_GAP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0i \in (A\_27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (ap (c\_2Ebool\_2E\_21 A\_27b) (\lambda V2v \in A\_27b. (a$

**Definition 11** We define  $c\_2EquantHeuristics\_2EGUESS\_FORALL\_POINT$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0i \in (A\_27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (ap (c\_2Ebool\_2E\_21 A\_27b) (\lambda V2fv \in A\_27b. (a$

**Definition 12** We define  $c\_2EquantHeuristics\_2EGUESS\_EXISTS\_POINT$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0i \in (A\_27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (ap (c\_2Ebool\_2E\_21 A\_27a) (\lambda V2fv \in A\_27a. (a$

**Definition 13** We define  $c\_2EquantHeuristics\_2EGUESS\_FORALL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0i \in (A\_27a)$

**Definition 14** We define  $c\_2EquantHeuristics\_2EGUESS\_EXISTS$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0i \in (A\_27a)$

**Definition 15** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. ((V0t1 = V1t2) \Rightarrow (V0t1 = V2t)))))$

**Definition 16** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. ((V0t1 = V1t2) \Rightarrow (V0t1 = V2t))))))$

Assume the following.

$$True \quad (2)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (4)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (5)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (9)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg (\neg (p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge (((\neg False) \Leftrightarrow True)))))) \quad (10)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in A_{27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(p V0A) \wedge (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (14)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27}))))))) \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow \\ & \quad \forall V0i \in (A_{27b}^{A_{27a}}).(\forall V1P \in (2^{A_{27b}}).(((p (ap (c_2EquantHeuristics_2EGUESS_EXISTS A_{27a} A_{27b}) V0i) V1P)) \Leftrightarrow \\ & \quad (ap V1P (ap V0i V3fv))))) \wedge ((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL A_{27a} A_{27b}) V0i) V1P)) \Leftrightarrow (\forall V4v \in A_{27b}.((\neg(p (ap V1P V4v)) \Rightarrow \\ & \quad (\exists V5fv \in A_{27a}.(\neg(p (ap V1P (ap V0i V5fv))))))) \wedge ((\forall V6i \in (A_{27b}^{A_{27a}}).(\forall V7P \in (2^{A_{27b}}).((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_POINT A_{27a} A_{27b}) V6i) V7P)) \Leftrightarrow (\forall V8fv \in A_{27a}.(p (ap V7P (ap V6i V8fv))))))) \wedge \\ & \quad ((\forall V9i \in (A_{27b}^{A_{27a}}).(\forall V10P \in (2^{A_{27b}}).((p (ap (c_2EquantHeuristics_2EGUESS_FORALL_POINT A_{27a} A_{27b}) V9i) V10P)) \Leftrightarrow (\forall V11fv \in A_{27a}.(\neg(p (ap V10P (ap V9i V11fv))))))) \wedge \\ & \quad ((\forall V12i \in (A_{27b}^{A_{27a}}).(\forall V13P \in (2^{A_{27b}}).((p (ap (c_2EquantHeuristics_2EGUESS_EXISTS_GAP A_{27a} A_{27b}) V12i) V13P)) \Leftrightarrow (\forall V14v \in A_{27b}.((p (ap V13P V14v)) \Rightarrow (\exists V15fv \in A_{27a}.(V14v = (ap V12i V15fv))))))) \wedge ((\forall V16i \in (A_{27b}^{A_{27a}}). \\ & \quad (\forall V17P \in (2^{A_{27b}}).((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL_GAP A_{27a} A_{27b}) V16i) V17P)) \Leftrightarrow (\forall V18v \in A_{27b}.((\neg(p (ap V17P V18v)) \Rightarrow (\exists V19fv \in A_{27a}.(V18v = (ap V16i V19fv))))))))))) \quad (16) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p)) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q))))) \quad (28)$$

### Theorem 1

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\
& \forall V0i \in (A_{27b}^{A_{27a}}).(\forall V1P \in (2^{A_{27b}}).(\forall V2Q \in \\
& (2^{A_{27b}}).(\forall V3iK \in A_{27b}.(\forall V4q \in 2.(\forall V5p \in \\
& 2.(((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& A_{27a} A_{27b}) V0i) (\lambda V6x \in A_{27b}.(ap V1P V6x)))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS \\
& A_{27a} A_{27b}) V0i) (\lambda V7x \in A_{27b}.(ap (ap c_2Ebool_2E_5C_2F (ap \\
& V1P V7x)) (ap V2Q V7x))))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& A_{27a} A_{27b}) V0i) (\lambda V8x \in A_{27b}.(ap V2Q V8x)))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS \\
& A_{27a} A_{27b}) V0i) (\lambda V9x \in A_{27b}.(ap (ap c_2Ebool_2E_5C_2F (ap \\
& V1P V9x)) (ap V2Q V9x))))))) \wedge (((((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS \\
& A_{27a} A_{27b}) V0i) (\lambda V10x \in A_{27b}.(ap V1P V10x)))) \wedge (p (ap (ap \\
& c_2EquantHeuristics_2EGUESS_EXISTS A_{27a} A_{27b}) V0i) (\lambda V11x \in \\
& A_{27b}.(ap V2Q V11x)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS \\
& A_{27a} A_{27b}) V0i) (\lambda V12x \in A_{27b}.(ap (ap c_2Ebool_2E_5C_2F (ap \\
& V1P V12x)) (ap V2Q V12x))))))) \wedge (((((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_GA \\
& A_{27a} A_{27b}) V0i) (\lambda V13x \in A_{27b}.(ap V1P V13x)))) \wedge (p (ap (ap \\
& c_2EquantHeuristics_2EGUESS_EXISTS_GAP A_{27a} A_{27b}) V0i) \\
& (\lambda V14x \in A_{27b}.(ap V2Q V14x)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_GA \\
& A_{27a} A_{27b}) V0i) (\lambda V15x \in A_{27b}.(ap (ap c_2Ebool_2E_5C_2F (ap \\
& V1P V15x)) (ap V2Q V15x))))))) \wedge (((((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& ty_2Eone_2Eone A_{27b}) (\lambda V16xxx \in ty_2Eone_2Eone.V3iK)) (\lambda V17x \in \\
& A_{27b}.(ap V1P V17x))) \wedge (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& ty_2Eone_2Eone A_{27b}) (\lambda V18xxx \in ty_2Eone_2Eone.V3iK)) (\lambda V19x \in \\
& A_{27b}.(ap V2Q V19x)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& ty_2Eone_2Eone A_{27b}) (\lambda V20xxx \in ty_2Eone_2Eone.V3iK)) (\lambda V21x \in \\
& A_{27b}.(ap (ap c_2Ebool_2E_5C_2F (ap V1P V21x)) (ap V2Q V21x)))))) \wedge \\
& (((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL A_{27a} A_{27b}) V0i) \\
& (\lambda V22x \in A_{27b}.(ap V1P V22x)))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& A_{27a} A_{27b}) V0i) (\lambda V23x \in A_{27b}.(ap (ap c_2Ebool_2E_5C_2F (ap \\
& V1P V23x)) V4q)))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& A_{27a} A_{27b}) V0i) (\lambda V24x \in A_{27b}.(ap V2Q V24x)))) \Rightarrow (p (ap (ap \\
& c_2EquantHeuristics_2EGUESS_FORALL A_{27a} A_{27b}) V0i) (\lambda V25x \in \\
& A_{27b}.(ap (ap c_2Ebool_2E_5C_2F V5p) (ap V2Q V25x)))))) \wedge (((p \\
& (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL A_{27a} A_{27b}) V0i) \\
& (\lambda V26x \in A_{27b}.(ap V1P V26x)))) \wedge (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& A_{27a} A_{27b}) V0i) (\lambda V27x \in A_{27b}.(ap V2Q V27x)))))) \Rightarrow (p (ap (ap \\
& (c_2EquantHeuristics_2EGUESS_FORALL A_{27a} A_{27b}) V0i) (\lambda V28x \in A_{27b}.(ap (ap \\
& c_2Ebool_2E_5C_2F (ap V1P V28x)) (ap V2Q V28x)))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& A_{27a} A_{27b}) V0i) (\lambda V29x \in A_{27b}.(ap V1P V29x)))) \Rightarrow (p (ap (ap \\
& c_2EquantHeuristics_2EGUESS_FORALL A_{27a} A_{27b}) V0i) (\lambda V30x \in A_{27b}.(ap (ap \\
& c_2Ebool_2E_5C_2F (ap V1P V30x)) (ap V2Q V30x)))))) \wedge ((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& A_{27a} A_{27b}) V0i) (\lambda V31x \in A_{27b}.(ap V2Q V31x)))) \Rightarrow (p (ap (ap \\
& c_2EquantHeuristics_2EGUESS_FORALL A_{27a} A_{27b}) V0i) (\lambda V32x \in A_{27b}.(ap (ap \\
& c_2Ebool_2E_5C_2F (ap V1P V32x)) (ap V2Q V32x)))))))))))))))))))))))
\end{aligned}$$