

thm_2EquantHeuristics_2EGUESS__RULES__EQUATION__EXISTS__GAP
 (TMFwpScdsaMb-
 vXV3wmQdvkT2b3nSryWzr25)

October 26, 2020

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 9 We define $c_2EquantHeuristics_2EGUESS_FORALL_GAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (2^{A_27a}).$

Definition 10 We define $c_2EquantHeuristics_2EGUESS_EXISTS_GAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2v \in A_27b.(ap$

Definition 11 We define $c_2EquantHeuristics_2EGUESS_FORALL_POINT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27a) (\lambda V2fv \in A_27b.(ap$

Definition 12 We define `c_2EquantHeuristics_2EGUESS_EXISTS_POINT` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27b}^{A_{.27a}}). \lambda V1P \in (2^{A_{.27b}}). (ap (c_2Ebool_2E_21 A_{.27a}) (\lambda V2fv \in A_{.27a}.$

Definition 13 We define `c_2EquantHeuristics_2EGUESS_FORALL` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27b}^{A_{.27a}}).$

Definition 14 We define `c_2EquantHeuristics_2EGUESS_EXISTS` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27b}^{A_{.27a}}).$

Definition 15 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \tag{2}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{.27a}. (p V0t)) \Leftrightarrow (p V0t))) \tag{3}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{4}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. ((V0x = V0x) \Leftrightarrow True)) \tag{5}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p\ (ap\ (\\
& \quad ap\ (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ V1P)) \Leftrightarrow \\
& \quad (\forall V2v \in A_27b. ((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A_27a. (p\ (\\
& \quad ap\ V1P\ (ap\ V0i\ V3fv)))))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_27a\ A_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V4v \in A_27b. ((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow \\
& \quad (\exists V5fv \in A_27a. (\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv)))))) \wedge ((\forall V6i \in \\
& \quad (A_27b^{A_27a}). (\forall V7P \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_27a\ A_27b)\ V6i)\ V7P)) \Leftrightarrow (\forall V8fv \in A_27a. (p\ (ap\ V7P\ (ap\ V6i\ V8fv)))))) \wedge \\
& \quad ((\forall V9i \in (A_27b^{A_27a}). (\forall V10P \in (2^{A_27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27a\ A_27b) \\
& \quad V9i)\ V10P)) \Leftrightarrow (\forall V11fv \in A_27a. (\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))) \wedge \\
& \quad ((\forall V12i \in (A_27b^{A_27a}). (\forall V13P \in (2^{A_27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b) \\
& \quad V12i)\ V13P)) \Leftrightarrow (\forall V14v \in A_27b. ((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in \\
& \quad A_27a. (V14v = (ap\ V12i\ V15fv)))))) \wedge (\forall V16i \in (A_27b^{A_27a}). \\
& \quad (\forall V17P \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27a\ A_27b)\ V16i)\ V17P)) \Leftrightarrow (\forall V18v \in A_27b. ((\neg(p\ (ap\ V17P\ V18v)) \Rightarrow \\
& \quad (\exists V19fv \in A_27a. (V18v = (ap\ V16i\ V19fv)))))))))
\end{aligned} \tag{9}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in A_27a. (\forall V1P \in (A_27b^{A_27a}). (\forall V2Q \in (\\
& \quad A_27b^{A_27a}). (((ap\ V1P\ V0i) = (ap\ V2Q\ V0i)) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_PO \\
& \quad ty_2Eone_2Eone\ A_27a)\ (\lambda V3xxx \in ty_2Eone_2Eone.V0i))\ (\lambda V4x \in \\
& \quad A_27a. (ap\ (ap\ (c_2Emin_2E_3D\ A_27b)\ (ap\ V1P\ V4x))\ (ap\ V2Q\ V4x))))))
\end{aligned}$$