thm_2EquantHeuristics_2EGUESS__RULES__EQUATION__FORAI (TMLmqeau1mWjFDk7J4xubZ8WiLYbqNF9CDd)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 6 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 9 *We define* c_2EquantHeuristics_2EGUESS__FORALL__GAP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in ($

Definition 10 *We define* c_2EquantHeuristics_2EGUESS__EXISTS__GAP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27b^{A\_27a}).\lambda V1P \in (2^{A\_27b}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27b)\ (\lambda V2v \in A\_27b.(a$

Definition 11 *We define* c_2EquantHeuristics_2EGUESS__FORALL__POINT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27b^{A\_27a}).\lambda V1P \in (2^{A\_27b}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27a)\ (\lambda V2fv \in A\_27$

Definition 12 *We define* c_2EquantHeuristics_2EGUESS__EXISTS__POINT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27b^{A\_27a}).\lambda V1P \in (2^{A\_27b}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27a)\ (\lambda V2fv \in A\_27a.$

Definition 13 *We define* c_2EquantHeuristics_2EGUESS__FORALL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27$

Definition 14 *We define* c_2EquantHeuristics_2EGUESS__EXISTS *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27$

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{5}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{7}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{8}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27b}).((((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V2v \in A\_27b.((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A\_27a.(p\ (ap\ V1P\ (ap\ V0i\ V3fv))))))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V4v \in A\_27b.((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow (\exists V5fv \in A\_27a.(\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv))))))))) \wedge ((\forall V6i \in (A\_27b^{A\_27a}).(\forall V7P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_POINT}\ A\_27a\ A\_27b)\ V6i)\ V7P)) \Leftrightarrow (\forall V8fv \in A\_27a.(p\ (ap\ V7P\ (ap\ V6i\ V8fv))))))) \wedge ((\forall V9i \in (A\_27b^{A\_27a}).(\forall V10P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_POINT}\ A\_27a\ A\_27b)\ V9i)\ V10P)) \Leftrightarrow (\forall V11fv \in A\_27a.(\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))))) \wedge ((\forall V12i \in (A\_27b^{A\_27a}).(\forall V13P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_GAP}\ A\_27a\ A\_27b)\ V12i)\ V13P)) \Leftrightarrow (\forall V14v \in A\_27b.((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in A\_27a.(V14v = (ap\ V12i\ V15fv)))))))) \wedge (\forall V16i \in (A\_27b^{A\_27a}). (\forall V17P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_GAP}\ A\_27a\ A\_27b)\ V16i)\ V17P)) \Leftrightarrow (\forall V18v \in A\_27b.((\neg(p\ (ap\ V17P\ V18v))) \Rightarrow (\exists V19fv \in A\_27a.(V18v = (ap\ V16i\ V19fv))))))))))))) \tag{10}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0i \in (A\_27b^{A\_27a}).(\forall V1P \in (A\_27c^{A\_27b}). (\forall V2Q \in (A\_27c^{A\_27b}).((\forall V3fv \in A\_27a.(\neg((ap\ V1P\ (ap\ V0i\ V3fv)) = (ap\ V2Q\ (ap\ V0i\ V3fv))))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_PO} A\_27a\ A\_27b)\ V0i)\ (\lambda V4x \in A\_27b.(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27c) (ap\ V1P\ V4x))\ (ap\ V2Q\ V4x)))))))))$$