

thm_2EquantHeuristics_2EGUESS__RULES__EXISTS
(TM-
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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A a))))$

Definition 4 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (ap V0P (ap (c_2Emin_2E_40 A a))))))$

Definition 6 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F))$

Definition 9 We define $c_2EquantHeuristics_2EGUESS_EXISTS_GAP$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0i \in (A-27a)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 10 We define $c_2EquantHeuristics_2EGUESS_EXISTS$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0i \in (A-27a)$

Definition 11 We define $c_2EquantHeuristics_2EGUESS_EXISTS_POINT$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0i \in (A-27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A-27a) (\lambda V2fv \in A-27a))$

Definition 12 We define $c_2EquantHeuristics_2EGUESS_FORALL_GAP$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0i \in (A-27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A-27b) (\lambda V2v \in A-27b.(ap (c_2Emin_2E_3D (2^2)) (\lambda V3x \in A-27b.V3x))))$

Definition 13 We define $\text{c_2EquantHeuristics_2EGUESS_FORALL}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27$

Definition 14 We define $\text{c_2EquantHeuristics_2EGUESS_FORALL_POINT}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (\text{c_2Ebool_2E_21 } A_27a) (\lambda V2fv \in A_27$

Definition 15 We define c_2Ebool_2E_2F_5C to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (2)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p (ap V0P V2x)))))) \quad (3)$$

Assume the following.

$$\begin{aligned} &\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ &\quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p (ap (ap \\ &\quad (\text{c_2EquantHeuristics_2EGUESS_EXISTS } A_27a A_27b) V0i) (\lambda V2x \in \\ &\quad A_27b.(ap \text{c_2Ebool_2E_7E } (ap V1P V2x)))))) \Leftrightarrow (p (ap (ap (\text{c_2EquantHeuristics_2EGUESS_FORALL} \\ &\quad A_27a A_27b) V0i) (\lambda V3x \in A_27b.(ap V1P V3x)))))) \wedge (((p (ap (ap \\ &\quad (\text{c_2EquantHeuristics_2EGUESS_FORALL } A_27a A_27b) V0i) (\lambda V4x \in \\ &\quad A_27b.(ap \text{c_2Ebool_2E_7E } (ap V1P V4x)))))) \Leftrightarrow (p (ap (ap (\text{c_2EquantHeuristics_2EGUESS_EXISTS} \\ &\quad A_27a A_27b) V0i) (\lambda V5x \in A_27b.(ap V1P V5x)))))) \wedge (((p (ap (ap \\ &\quad (\text{c_2EquantHeuristics_2EGUESS_EXISTS_GAP } A_27a A_27b) V0i) \\ &\quad (\lambda V6x \in A_27b.(ap \text{c_2Ebool_2E_7E } (ap V1P V6x)))))) \Leftrightarrow (p (ap (ap \\ &\quad (\text{c_2EquantHeuristics_2EGUESS_FORALL_GAP } A_27a A_27b) V0i) \\ &\quad (\lambda V7x \in A_27b.(ap V1P V7x)))))) \wedge (((p (ap (ap (\text{c_2EquantHeuristics_2EGUESS_FORALL_GAP} \\ &\quad A_27a A_27b) V0i) (\lambda V8x \in A_27b.(ap \text{c_2Ebool_2E_7E } (ap V1P V8x)))))) \Leftrightarrow \\ &\quad (p (ap (ap (\text{c_2EquantHeuristics_2EGUESS_EXISTS_GAP } A_27a A_27b) \\ &\quad V0i) (\lambda V9x \in A_27b.(ap V1P V9x)))))) \wedge (((p (ap (ap (\text{c_2EquantHeuristics_2EGUESS_EXISTS_POIN} \\ &\quad A_27a A_27b) V0i) (\lambda V10x \in A_27b.(ap \text{c_2Ebool_2E_7E } (ap V1P V10x)))))) \Leftrightarrow \\ &\quad (p (ap (ap (\text{c_2EquantHeuristics_2EGUESS_FORALL_POINT } A_27a \\ &\quad A_27b) V0i) (\lambda V11x \in A_27b.(ap V1P V11x)))))) \wedge (((p (ap (ap (\text{c_2EquantHeuristics_2EGUESS_FORALL} \\ &\quad A_27a A_27b) V0i) (\lambda V12x \in A_27b.(ap \text{c_2Ebool_2E_7E } (ap V1P V12x)))))) \Leftrightarrow \\ &\quad (p (ap (ap (\text{c_2EquantHeuristics_2EGUESS_EXISTS_POINT } A_27a \\ &\quad A_27b) V0i) (\lambda V13x \in A_27b.(ap V1P V13x)))))))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.\text{nonempty } A.27a \Rightarrow \forall A.27b.\text{nonempty } A.27b \Rightarrow \forall A.27c. \\
& \text{nonempty } A.27c \Rightarrow (\forall V0i \in (A.27c^{A.27b}). (\forall V1P \in ((2^{A.27a})^{A.27c}). \\
& (\forall V2iK \in A.27c. ((\forall V3y \in A.27a. (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL_POINT \\
& A.27b A.27c) V0i) (\lambda V4x \in A.27c. (ap (ap V1P V4x) V3y)))))) \Rightarrow (p (\\
& ap (ap (c.2EquantHeuristics_2EGUESS_FORALL_POINT A.27b A.27c) \\
& V0i) (\lambda V5x \in A.27c. (ap (c.2Ebool_2E.21 A.27a) (\lambda V6y \in A.27a. \\
& (ap (ap V1P V5x) V6y)))))) \wedge ((\forall V7y \in A.27a. (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL \\
& A.27b A.27c) V0i) (\lambda V8x \in A.27c. (ap (ap V1P V8x) V7y)))))) \Rightarrow (p (\\
& ap (ap (c.2EquantHeuristics_2EGUESS_FORALL A.27b A.27c) V0i) \\
& (\lambda V9x \in A.27c. (ap (c.2Ebool_2E.21 A.27a) (\lambda V10y \in A.27a. \\
& (ap (ap V1P V9x) V10y)))))) \wedge ((\forall V11y \in A.27a. (p (ap (ap (\\
& c.2EquantHeuristics_2EGUESS_FORALL_GAP A.27b A.27c) V0i) \\
& (\lambda V12x \in A.27c. (ap (ap V1P V12x) V11y)))))) \Rightarrow (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL_GAP \\
& A.27b A.27c) V0i) (\lambda V13x \in A.27c. (ap (c.2Ebool_2E.21 A.27a) \\
& (\lambda V14y \in A.27a. (ap (ap V1P V13x) V14y)))))) \wedge ((\forall V15y \in \\
& A.27a. (p (ap (ap (c.2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& A.27b A.27c) V0i) (\lambda V16x \in A.27c. (ap (ap V1P V16x) V15y)))))) \Rightarrow \\
& (p (ap (ap (c.2EquantHeuristics_2EGUESS_EXISTS_POINT A.27b \\
& A.27c) V0i) (\lambda V17x \in A.27c. (ap (c.2Ebool_2E.21 A.27a) (\lambda V18y \in \\
& A.27a. (ap (ap V1P V17x) V18y)))))) \wedge ((\forall V19y \in A.27a. (p \\
& (ap (ap (c.2EquantHeuristics_2EGUESS_EXISTS ty_2Eone_2Eone \\
& A.27c) (\lambda V20xxx \in ty_2Eone_2Eone.V2iK)) (\lambda V21x \in A.27c. \\
& (ap (ap V1P V21x) V19y)))))) \Rightarrow (p (ap (ap (c.2EquantHeuristics_2EGUESS_EXISTS \\
& ty_2Eone_2Eone A.27c) (\lambda V22xxx \in ty_2Eone_2Eone.V2iK)) (\lambda V23x \in \\
& A.27c. (ap (c.2Ebool_2E.21 A.27a) (\lambda V24y \in A.27a. (ap (ap V1P \\
& V23x) V24y)))))) \wedge ((\forall V25y \in A.27a. (p (ap (ap (c.2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& A.27b A.27c) V0i) (\lambda V26x \in A.27c. (ap (ap V1P V26x) V25y)))))) \Rightarrow \\
& (p (ap (ap (c.2EquantHeuristics_2EGUESS_EXISTS_GAP A.27b A.27c) \\
& V0i) (\lambda V27x \in A.27c. (ap (c.2Ebool_2E.21 A.27a) (\lambda V28y \in A.27a. \\
& (ap (ap V1P V27x) V28y)))))))))
\end{aligned} \tag{5}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0i \in (A_27c^{A_27b}). (\forall V1P \in ((2^{A_27a})^{A_27c}). \\
& (\forall V2iK \in A_27c. (((\forall V3y \in A_27a. (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_27b\ A_27c)\ V0i)\ (\lambda V4x \in A_27c. (ap\ (ap\ V1P\ V4x)\ V3y)))))) \Rightarrow (p\ (\\
& \quad ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT\ A_27b\ A_27c) \\
& \quad V0i)\ (\lambda V5x \in A_27c. (ap\ (c_2Ebool_2E_3F\ A_27a)\ (\lambda V6y \in A_27a. \\
& (ap\ (ap\ V1P\ V5x)\ V6y)))))) \wedge (((\forall V7y \in A_27a. (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS \\
& \quad A_27b\ A_27c)\ V0i)\ (\lambda V8x \in A_27c. (ap\ (ap\ V1P\ V8x)\ V7y)))))) \Rightarrow (p\ (\\
& \quad ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27b\ A_27c)\ V0i) \\
& \quad (\lambda V9x \in A_27c. (ap\ (c_2Ebool_2E_3F\ A_27a)\ (\lambda V10y \in A_27a. \\
& \quad (ap\ (ap\ V1P\ V9x)\ V10y)))))) \wedge (((\forall V11y \in A_27a. (p\ (ap\ (ap\ (\\
& \quad c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27b\ A_27c)\ V0i) \\
& (\lambda V12x \in A_27c. (ap\ (ap\ V1P\ V12x)\ V11y)))))) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GA \\
& \quad A_27b\ A_27c)\ V0i)\ (\lambda V13x \in A_27c. (ap\ (c_2Ebool_2E_3F\ A_27a) \\
& \quad (\lambda V14y \in A_27a. (ap\ (ap\ V1P\ V13x)\ V14y)))))) \wedge (((\forall V15y \in \\
& \quad A_27a. (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT \\
& \quad A_27b\ A_27c)\ V0i)\ (\lambda V16x \in A_27c. (ap\ (ap\ V1P\ V16x)\ V15y)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27b \\
& \quad A_27c)\ V0i)\ (\lambda V17x \in A_27c. (ap\ (c_2Ebool_2E_3F\ A_27a)\ (\lambda V18y \in \\
& \quad A_27a. (ap\ (ap\ V1P\ V17x)\ V18y)))))) \wedge (((\forall V19y \in A_27a. (p \\
& \quad (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL\ ty_2Eone_2Eone \\
& \quad A_27c)\ (\lambda V20xxx \in ty_2Eone_2Eone.V2iK))\ (\lambda V21x \in A_27c. \\
& (ap\ (ap\ V1P\ V21x)\ V19y)))))) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad ty_2Eone_2Eone\ A_27c)\ (\lambda V22xxx \in ty_2Eone_2Eone.V2iK))\ (\lambda V23x \in \\
& \quad A_27c. (ap\ (c_2Ebool_2E_3F\ A_27a)\ (\lambda V24y \in A_27a. (ap\ (ap\ V1P \\
& \quad V23x)\ V24y)))))) \wedge (((\forall V25y \in A_27a. (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27b\ A_27c)\ V0i)\ (\lambda V26x \in A_27c. (ap\ (ap\ V1P\ V26x)\ V25y)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP\ A_27b\ A_27c) \\
& \quad V0i)\ (\lambda V27x \in A_27c. (ap\ (c_2Ebool_2E_3F\ A_27a)\ (\lambda V28y \in A_27a. \\
& \quad (ap\ (ap\ V1P\ V27x)\ V28y)))))))))
\end{aligned}$$