

thm_2EquantHeuristics_2EGUESS__RULES__EXISTS_____NEW__P (TMH9wtpPsJuKX6XishfGB8Afa6tnehg3qNV)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A \text{ (ap } P \ x))$

Definition 4 We define `c_2Ebool_2E_T` to be $(\text{ap (ap (c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap (c_2Emin_2E_3D } (2^{A-27a}).$

Definition 6 We define `c_2EquantHeuristics_2EGUESS__EXISTS__POINT` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (\text{ap (c_2Ebool_2E_21 } A. 27a) (\lambda V2fv \in A. 27a.$

Definition 7 We define `c_2Ebool_2E_F` to be $(\text{ap (c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t)).$

Definition 8 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type $\iota.$

Definition 9 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap (ap c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2E_F$

Definition 10 We define `c_2EquantHeuristics_2EGUESS__EXISTS` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}).$

Definition 11 We define `c_2EquantHeuristics_2EGUESS__EXISTS__GAP` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (\text{ap (c_2Ebool_2E_21 } A. 27b) (\lambda V2v \in A. 27b. (\text{ap } P \ v)$

Definition 12 We define `c_2EquantHeuristics_2EGUESS__FORALL__GAP` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}). \lambda V1P \in (2^{A-27b}). (\text{ap (c_2Ebool_2E_21 } A. 27b) (\lambda V2v \in A. 27b. (\text{ap } P \ v)$

Definition 13 We define `c_2EquantHeuristics_2EGUESS__FORALL` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0i \in (A. 27b^{A-27a}).$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (3)$$

Definition 14 We define $c_2EquantHeuristics_2EGUESS_FORALL_POINT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap\ (c_2Ebool_2E_21\ A_27a)\ (\lambda V2fv \in A_27b))$

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2).(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p\ (ap\ V0P\ V2x))))))) \quad (5)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& \quad \forall V0i \in (A_{.27b}^{A_{.27a}}). (\forall V1P \in (2^{A_{.27b}}). ((p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_EXISTS\ A_{.27a}\ A_{.27b})\ V0i)\ (\lambda V2x \in \\
& \quad A_{.27b}.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V2x)))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_{.27a}\ A_{.27b})\ V0i)\ (\lambda V3x \in A_{.27b}.(ap\ V1P\ V3x)))) \wedge ((p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_FORALL\ A_{.27a}\ A_{.27b})\ V0i)\ (\lambda V4x \in \\
& \quad A_{.27b}.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V4x)))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS \\
& \quad A_{.27a}\ A_{.27b})\ V0i)\ (\lambda V5x \in A_{.27b}.(ap\ V1P\ V5x)))) \wedge ((p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_{.27a}\ A_{.27b})\ V0i) \\
& \quad (\lambda V6x \in A_{.27b}.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V6x)))) \Leftrightarrow (p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_FORALL_GAP\ A_{.27a}\ A_{.27b})\ V0i) \\
& \quad (\lambda V7x \in A_{.27b}.(ap\ V1P\ V7x)))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_{.27a}\ A_{.27b})\ V0i)\ (\lambda V8x \in A_{.27b}.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V8x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_{.27a}\ A_{.27b}) \\
& \quad V0i)\ (\lambda V9x \in A_{.27b}.(ap\ V1P\ V9x)))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POIN \\
& \quad A_{.27a}\ A_{.27b})\ V0i)\ (\lambda V10x \in A_{.27b}.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V10x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_{.27a} \\
& \quad A_{.27b})\ V0i)\ (\lambda V11x \in A_{.27b}.(ap\ V1P\ V11x)))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_ \\
& \quad A_{.27a}\ A_{.27b})\ V0i)\ (\lambda V12x \in A_{.27b}.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V12x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT\ A_{.27a} \\
& \quad A_{.27b})\ V0i)\ (\lambda V13x \in A_{.27b}.(ap\ V1P\ V13x)))))))))
\end{aligned}$$

(6)

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0iy \in ((A_27c^{A_27b})^{A_27a}). (\forall V1P \in \\
& ((2^{A_27a})^{A_27c}). (((\forall V2y \in A_27a. (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT \\
& \quad A_27b\ A_27c)\ (ap\ V0iy\ V2y))\ (\lambda V3x \in A_27c. (ap\ (ap\ V1P\ V3x)\ V2y)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)\ A_27c)\ (\lambda V4fv \in (ty_2Epair_2Eprod\ A_27a\ A_27b). \\
& \quad (ap\ (ap\ V0iy\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V4fv))\ (ap\ (c_2Epair_2ESND \\
& \quad A_27a\ A_27b)\ V4fv))))\ (\lambda V5x \in A_27c. (ap\ (c_2Ebool_2E_21\ A_27a) \\
& \quad (\lambda V6y \in A_27a. (ap\ (ap\ V1P\ V5x)\ V6y)))))) \wedge (((\forall V7y \in A_27a. \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL\ A_27b\ A_27c) \\
& \quad (ap\ V0iy\ V7y))\ (\lambda V8x \in A_27c. (ap\ (ap\ V1P\ V8x)\ V7y)))))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ (c_2EquantHeuristics_2EGUESS_FORALL\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)\ A_27c)\ (\lambda V9fv \in (ty_2Epair_2Eprod\ A_27a\ A_27b). \\
& \quad (ap\ (ap\ V0iy\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V9fv))\ (ap\ (c_2Epair_2ESND \\
& \quad A_27a\ A_27b)\ V9fv))))\ (\lambda V10x \in A_27c. (ap\ (c_2Ebool_2E_21\ A_27a) \\
& \quad (\lambda V11y \in A_27a. (ap\ (ap\ V1P\ V10x)\ V11y)))))) \wedge (((\forall V12y \in \\
& \quad A_27a. (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27b\ A_27c)\ (ap\ V0iy\ V12y))\ (\lambda V13x \in A_27c. (ap\ (ap\ V1P\ V13x)\ V12y)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)\ A_27c)\ (\lambda V14fv \in (ty_2Epair_2Eprod\ A_27a\ A_27b). \\
& \quad (ap\ (ap\ V0iy\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V14fv))\ (ap\ (c_2Epair_2ESND \\
& \quad A_27a\ A_27b)\ V14fv))))\ (\lambda V15x \in A_27c. (ap\ (c_2Ebool_2E_21\ A_27a) \\
& \quad (\lambda V16y \in A_27a. (ap\ (ap\ V1P\ V15x)\ V16y)))))) \wedge (((\forall V17y \in \\
& \quad A_27a. (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad A_27b\ A_27c)\ (ap\ V0iy\ V17y))\ (\lambda V18x \in A_27c. (ap\ (ap\ V1P\ V18x)\ V17y)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)\ A_27c)\ (\lambda V19fv \in (ty_2Epair_2Eprod\ A_27a\ A_27b). \\
& \quad (ap\ (ap\ V0iy\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V19fv))\ (ap\ (c_2Epair_2ESND \\
& \quad A_27a\ A_27b)\ V19fv))))\ (\lambda V20x \in A_27c. (ap\ (c_2Ebool_2E_21\ A_27a) \\
& \quad (\lambda V21y \in A_27a. (ap\ (ap\ V1P\ V20x)\ V21y)))))))))
\end{aligned}
\tag{7}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0iy \in ((A.27c^{A.27b})^{A.27a}). (\forall V1P \in \\
& ((2^{A.27a})^{A.27c}). ((\forall V2y \in A.27a. (p\ (ap\ (ap\ (c.2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A.27b\ A.27c)\ (ap\ V0iy\ V2y))\ (\lambda V3x \in A.27c. (ap\ (ap\ V1P\ V3x)\ V2y)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics_2EGUESS_EXISTS_POINT\ (ty_2Epair_2Eprod \\
& \quad A.27a\ A.27b)\ A.27c)\ (\lambda V4fv \in (ty_2Epair_2Eprod\ A.27a\ A.27b). \\
& \quad (ap\ (ap\ V0iy\ (ap\ (c.2Epair_2EFST\ A.27a\ A.27b)\ V4fv))\ (ap\ (c.2Epair_2ESND \\
& \quad A.27a\ A.27b)\ V4fv))))\ (\lambda V5x \in A.27c. (ap\ (c.2Ebool_2E_3F\ A.27a) \\
& \quad (\lambda V6y \in A.27a. (ap\ (ap\ V1P\ V5x)\ V6y)))))) \wedge ((\forall V7y \in A.27a. \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics_2EGUESS_EXISTS\ A.27b\ A.27c) \\
& \quad (ap\ V0iy\ V7y))\ (\lambda V8x \in A.27c. (ap\ (ap\ V1P\ V8x)\ V7y)))))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ (c.2EquantHeuristics_2EGUESS_EXISTS\ (ty_2Epair_2Eprod \\
& \quad A.27a\ A.27b)\ A.27c)\ (\lambda V9fv \in (ty_2Epair_2Eprod\ A.27a\ A.27b). \\
& \quad (ap\ (ap\ V0iy\ (ap\ (c.2Epair_2EFST\ A.27a\ A.27b)\ V9fv))\ (ap\ (c.2Epair_2ESND \\
& \quad A.27a\ A.27b)\ V9fv))))\ (\lambda V10x \in A.27c. (ap\ (c.2Ebool_2E_3F\ A.27a) \\
& \quad (\lambda V11y \in A.27a. (ap\ (ap\ V1P\ V10x)\ V11y)))))) \wedge ((\forall V12y \in \\
& \quad A.27a. (p\ (ap\ (ap\ (c.2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad A.27b\ A.27c)\ (ap\ V0iy\ V12y))\ (\lambda V13x \in A.27c. (ap\ (ap\ V1P\ V13x)\ V12y)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics_2EGUESS_EXISTS_GAP\ (ty_2Epair_2Eprod \\
& \quad A.27a\ A.27b)\ A.27c)\ (\lambda V14fv \in (ty_2Epair_2Eprod\ A.27a\ A.27b). \\
& \quad (ap\ (ap\ V0iy\ (ap\ (c.2Epair_2EFST\ A.27a\ A.27b)\ V14fv))\ (ap\ (c.2Epair_2ESND \\
& \quad A.27a\ A.27b)\ V14fv))))\ (\lambda V15x \in A.27c. (ap\ (c.2Ebool_2E_3F\ A.27a) \\
& \quad (\lambda V16y \in A.27a. (ap\ (ap\ V1P\ V15x)\ V16y)))))) \wedge ((\forall V17y \in \\
& \quad A.27a. (p\ (ap\ (ap\ (c.2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A.27b\ A.27c)\ (ap\ V0iy\ V17y))\ (\lambda V18x \in A.27c. (ap\ (ap\ V1P\ V18x)\ V17y)))))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics_2EGUESS_FORALL_GAP\ (ty_2Epair_2Eprod \\
& \quad A.27a\ A.27b)\ A.27c)\ (\lambda V19fv \in (ty_2Epair_2Eprod\ A.27a\ A.27b). \\
& \quad (ap\ (ap\ V0iy\ (ap\ (c.2Epair_2EFST\ A.27a\ A.27b)\ V19fv))\ (ap\ (c.2Epair_2ESND \\
& \quad A.27a\ A.27b)\ V19fv))))\ (\lambda V20x \in A.27c. (ap\ (c.2Ebool_2E_3F\ A.27a) \\
& \quad (\lambda V21y \in A.27a. (ap\ (ap\ V1P\ V20x)\ V21y)))))))))
\end{aligned}$$