

thm_2EquantHeuristics_2EGUESS_RULES_EXISTS_____NEW__F
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pDUD5QR7z3WyD14d3P4o83Gi)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27$

Definition 6 We define $c_2EquantHeuristics_2EGUESS_EXISTS_POINT$ to be
 $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A_27a) (\lambda V2fv \in A_27a.$

Definition 7 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F$

Definition 10 We define $c_2EquantHeuristics_2EGUESS_EXISTS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27$

Definition 11 We define $c_2EquantHeuristics_2EGUESS_EXISTS_GAP$ to be
 $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2v \in A_27b.(a$

Definition 12 We define $c_2EquantHeuristics_2EGUESS_FORALL_GAP$ to be
 $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2v \in A_27b.(a$

Definition 13 We define $c_2EquantHeuristics_2EGUESS_FORALL$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (1)$$

Definition 14 We define `c_2EquantHeuristics_2EGUESS_FORALL_POINT` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (ap\ (c_2Ebool_2E_21\ A_27a)\ (\lambda V2fv \in A_27b))$

Definition 15 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2))))$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (2)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p\ (ap\ (ap \\ & \quad (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ (\lambda V2x \in \\ & \quad A_27b. (ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V2x)))))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\ & \quad A_27a\ A_27b)\ V0i)\ (\lambda V3x \in A_27b. (ap\ V1P\ V3x)))))) \wedge (((p\ (ap\ (ap \\ & \quad (c_2EquantHeuristics_2EGUESS_FORALL\ A_27a\ A_27b)\ V0i)\ (\lambda V4x \in \\ & \quad A_27b. (ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V4x)))))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS \\ & \quad A_27a\ A_27b)\ V0i)\ (\lambda V5x \in A_27b. (ap\ V1P\ V5x)))))) \wedge (((p\ (ap\ (ap \\ & \quad (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b)\ V0i) \\ & \quad (\lambda V6x \in A_27b. (ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V6x)))))) \Leftrightarrow (p\ (ap\ (ap \\ & \quad (c_2EquantHeuristics_2EGUESS_FORALL_GAP\ A_27a\ A_27b)\ V0i) \\ & \quad (\lambda V7x \in A_27b. (ap\ V1P\ V7x)))))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\ & \quad A_27a\ A_27b)\ V0i)\ (\lambda V8x \in A_27b. (ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V8x)))))) \Leftrightarrow \\ & \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b) \\ & \quad V0i)\ (\lambda V9x \in A_27b. (ap\ V1P\ V9x)))))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\ & \quad A_27a\ A_27b)\ V0i)\ (\lambda V10x \in A_27b. (ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V10x)))))) \Leftrightarrow \\ & \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27a \\ & \quad A_27b)\ V0i)\ (\lambda V11x \in A_27b. (ap\ V1P\ V11x)))))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT \\ & \quad A_27a\ A_27b)\ V0i)\ (\lambda V12x \in A_27b. (ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V12x)))))) \Leftrightarrow \\ & \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT\ A_27a \\ & \quad A_27b)\ V0i)\ (\lambda V13x \in A_27b. (ap\ V1P\ V13x)))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27c.nonempty\ A.27c \Rightarrow (\\
& \quad \forall V0i \in (A.27c^{A.27a}). (\forall V1P \in ((2^{A.27a})^{A.27c}). (\\
& \quad (\forall V2y \in A.27a. (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL_POINT \\
& \quad \quad ty_2Eone_2Eone\ A.27c) (\lambda V3xxx \in ty_2Eone_2Eone. (ap\ V0i\ V2y))) \\
& \quad (\lambda V4x \in A.27c. (ap (ap\ V1P\ V4x)\ V2y)))))) \Rightarrow (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL_POINT \\
& \quad \quad A.27a\ A.27c)\ V0i) (\lambda V5x \in A.27c. (ap (c.2Ebool_2E_21\ A.27a) (\\
& \quad \quad \lambda V6y \in A.27a. (ap (ap\ V1P\ V5x)\ V6y)))))) \wedge (((\forall V7y \in A.27a. \\
& \quad (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL\ ty_2Eone_2Eone \\
& \quad \quad A.27c) (\lambda V8xxx \in ty_2Eone_2Eone. (ap\ V0i\ V7y))) (\lambda V9x \in A.27c. \\
& \quad (ap (ap\ V1P\ V9x)\ V7y)))))) \Rightarrow (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL \\
& \quad \quad A.27a\ A.27c)\ V0i) (\lambda V10x \in A.27c. (ap (c.2Ebool_2E_21\ A.27a) \\
& \quad \quad (\lambda V11y \in A.27a. (ap (ap\ V1P\ V10x)\ V11y)))))) \wedge (((\forall V12y \in \\
& \quad A.27a. (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad \quad ty_2Eone_2Eone\ A.27c) (\lambda V13xxx \in ty_2Eone_2Eone. (ap\ V0i\ V12y))) \\
& \quad (\lambda V14x \in A.27c. (ap (ap\ V1P\ V14x)\ V12y)))))) \Rightarrow (p (ap (ap (c.2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad \quad A.27a\ A.27c)\ V0i) (\lambda V15x \in A.27c. (ap (c.2Ebool_2E_21\ A.27a) \\
& \quad \quad (\lambda V16y \in A.27a. (ap (ap\ V1P\ V15x)\ V16y)))))) \wedge (((\forall V17y \in \\
& \quad A.27a. (p (ap (ap (c.2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad \quad ty_2Eone_2Eone\ A.27c) (\lambda V18xxx \in ty_2Eone_2Eone. (ap\ V0i\ V17y))) \\
& \quad (\lambda V19x \in A.27c. (ap (ap\ V1P\ V19x)\ V17y)))))) \Rightarrow (p (ap (ap (c.2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad \quad A.27a\ A.27c)\ V0i) (\lambda V20x \in A.27c. (ap (c.2Ebool_2E_21\ A.27a) \\
& \quad \quad (\lambda V21y \in A.27a. (ap (ap\ V1P\ V20x)\ V21y)))))))))
\end{aligned}$$

(5)

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27c}. \text{nonempty } A_{.27c} \Rightarrow (\\
& \quad \forall V0i \in (A_{.27c}^{A_{.27a}}). (\forall V1P \in ((2^{A_{.27a}})^{A_{.27c}}). ((\\
& (\forall V2y \in A_{.27a}. (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad ty_2Eone_2Eone A_{.27c}) (\lambda V3xxx \in ty_2Eone_2Eone. (ap V0i V2y))) \\
& (\lambda V4x \in A_{.27c}. (ap (ap V1P V4x) V2y)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_{.27a} A_{.27c}) V0i) (\lambda V5x \in A_{.27c}. (ap (c_2Ebool_2E_3F A_{.27a}) (\\
& \quad \lambda V6y \in A_{.27a}. (ap (ap V1P V5x) V6y)))))) \wedge ((\forall V7y \in A_{.27a}. \\
& \quad (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS ty_2Eone_2Eone \\
& \quad A_{.27c}) (\lambda V8xxx \in ty_2Eone_2Eone. (ap V0i V7y))) (\lambda V9x \in A_{.27c}. \\
& \quad (ap (ap V1P V9x) V7y)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS \\
& \quad A_{.27a} A_{.27c}) V0i) (\lambda V10x \in A_{.27c}. (ap (c_2Ebool_2E_3F A_{.27a}) \\
& \quad (\lambda V11y \in A_{.27a}. (ap (ap V1P V10x) V11y)))))) \wedge ((\forall V12y \in \\
& \quad A_{.27a}. (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad ty_2Eone_2Eone A_{.27c}) (\lambda V13xxx \in ty_2Eone_2Eone. (ap V0i V12y))) \\
& (\lambda V14x \in A_{.27c}. (ap (ap V1P V14x) V12y)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad A_{.27a} A_{.27c}) V0i) (\lambda V15x \in A_{.27c}. (ap (c_2Ebool_2E_3F A_{.27a}) \\
& \quad (\lambda V16y \in A_{.27a}. (ap (ap V1P V15x) V16y)))))) \wedge ((\forall V17y \in \\
& \quad A_{.27a}. (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad ty_2Eone_2Eone A_{.27c}) (\lambda V18xxx \in ty_2Eone_2Eone. (ap V0i V17y))) \\
& (\lambda V19x \in A_{.27c}. (ap (ap V1P V19x) V17y)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_{.27a} A_{.27c}) V0i) (\lambda V20x \in A_{.27c}. (ap (c_2Ebool_2E_3F A_{.27a}) \\
& \quad (\lambda V21y \in A_{.27a}. (ap (ap V1P V20x) V21y))))))))))
\end{aligned}$$