

thm_2EquantHeuristics_2EGUESS__RULES__FORALL
(TM-
cPR1tKkTAUANtC6Mo4Ba5ddQ5FZJnmeEv)

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Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 1 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2E2 to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E3F to be $\lambda A.\lambda P \in 2^A.\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ A\ P)))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A.\lambda P \in 2^A.\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ V0P))$

Definition 6 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow q)$ of type ι .

Definition 8 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E2F))$

Definition 9 We define $c_2EquantHeuristics_2EGUESS_FORALL_GAP$ to be $\lambda A.\lambda P \in 2^A.\lambda V0i \in A.V0i \in P$

Definition 10 We define $c_2EquantHeuristics_2EGUESS_EXISTS_GAP$ to be $\lambda A.\lambda P \in 2^A.\lambda V0i \in A.V0i \in P$

Definition 11 We define $c_2EquantHeuristics_2EGUESS_FORALL_POINT$ to be $\lambda A.\lambda P \in 2^A.\lambda V0i \in A.V0i \in P$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (11)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\neg(\forall V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p (ap V0P V2x)))))) \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\forall V2x \in A.27a. ((p (ap V0P V2x)) \Rightarrow (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (14)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27}))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (15)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p\ (ap\ (\\
& \quad ap\ (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ V1P))) \Leftrightarrow \\
& \quad (\forall V2v \in A_27b. ((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A_27a. (p\ (\\
& \quad ap\ V1P\ (ap\ V0i\ V3fv)))))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_27a\ A_27b)\ V0i)\ V1P))) \Leftrightarrow (\forall V4v \in A_27b. ((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow \\
& \quad (\exists V5fv \in A_27a. (\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv)))))) \wedge ((\forall V6i \in \\
& \quad (A_27b^{A_27a}). (\forall V7P \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_27a\ A_27b)\ V6i)\ V7P))) \Leftrightarrow (\forall V8fv \in A_27a. (p\ (ap\ V7P\ (ap\ V6i\ V8fv)))))) \wedge \\
& \quad ((\forall V9i \in (A_27b^{A_27a}). (\forall V10P \in (2^{A_27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27a\ A_27b) \\
& \quad V9i)\ V10P))) \Leftrightarrow (\forall V11fv \in A_27a. (\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))) \wedge \\
& \quad ((\forall V12i \in (A_27b^{A_27a}). (\forall V13P \in (2^{A_27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b) \\
& \quad V12i)\ V13P))) \Leftrightarrow (\forall V14v \in A_27b. ((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in \\
& \quad A_27a. (V14v = (ap\ V12i\ V15fv)))))) \wedge (\forall V16i \in (A_27b^{A_27a}). \\
& \quad (\forall V17P \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27a\ A_27b)\ V16i)\ V17P))) \Leftrightarrow (\forall V18v \in A_27b. ((\neg(p\ (ap\ V17P\ V18v))) \Rightarrow \\
& \quad (\exists V19fv \in A_27a. (V18v = (ap\ V16i\ V19fv))))))))) \\
& \hspace{15em} (16)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (20)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee \neg(p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge ((\\
& \neg(p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow \neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge (\neg(p \vee 1q) \vee \neg(p \vee 0p))))))
\end{aligned} \tag{25}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p))) \tag{26}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow \neg(p \vee 1q))) \tag{27}$$

