

thm_2EquantHeuristics_2EGUESS_RULES_IMP (TMF7E5QgGxR4nSoYJ87bSSSUoaVkyPcouQu)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 10 We define $c_2EquantHeuristics_2EGUESS_FORALL_GAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2v \in A_27b.(ap$

Definition 11 We define $c_2EquantHeuristics_2EGUESS_FORALL_POINT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_21 A_27a) (\lambda V2fv \in A_27b.(ap$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 12 We define $c_2EquantHeuristics_2EGUESS_FORALL$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap (c_2Ebool_2E_3F A_27a) (\lambda V2fv \in A_27b.(ap$

Definition 13 We define `c_2EquantHeuristics_2EGUESS_EXISTS_GAP` to be
 $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27b}^{A_{.27a}}). \lambda V1P \in (2^{A_{.27b}}). (ap (c_{.2Ebool_2E_21} A_{.27b}) (\lambda V2v \in A_{.27b}. (a$

Definition 14 We define `c_2EquantHeuristics_2EGUESS_EXISTS` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27b}^{A_{.27a}}).$

Definition 15 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_{.2Ebool_2E_21} 2) (\lambda V2t \in 2. ($

Definition 16 We define `c_2EquantHeuristics_2EGUESS_EXISTS_POINT` to be
 $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27b}^{A_{.27a}}). \lambda V1P \in (2^{A_{.27b}}). (ap (c_{.2Ebool_2E_21} A_{.27a}) (\lambda V2fv \in A_{.27a}. ($

Definition 17 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_{.2Ebool_2E_21} 2) (\lambda V2t \in 2. ($

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (2)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg (p V0A)) \vee (p V1B)))))) \quad (3)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (4)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (5)$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow \forall A_{.27b}. nonempty A_{.27b} \Rightarrow \forall A_{.27c}. nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}). (\forall V1g \in (A_{.27a}^{A_{.27c}}). (\forall V2x \in A_{.27c}. ((ap (ap (ap (c_{.2Ecombin_2Eo} A_{.27c} A_{.27b} A_{.27a}) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (6)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0i \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27b}). ((p\ (ap\ (ap \\
& \quad (c.2EquantHeuristics.2EGUESS_EXISTS\ A.27a\ A.27b)\ V0i)\ (\lambda V2x \in \\
& \quad A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V2x)))) \Leftrightarrow (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_FORALL \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V3x \in A.27b.(ap\ V1P\ V3x)))) \wedge ((p\ (ap\ (ap \\
& \quad (c.2EquantHeuristics.2EGUESS_FORALL\ A.27a\ A.27b)\ V0i)\ (\lambda V4x \in \\
& \quad A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V4x)))) \Leftrightarrow (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_EXISTS \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V5x \in A.27b.(ap\ V1P\ V5x)))) \wedge ((p\ (ap\ (ap \\
& \quad (c.2EquantHeuristics.2EGUESS_EXISTS_GAP\ A.27a\ A.27b)\ V0i) \\
& \quad (\lambda V6x \in A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V6x)))) \Leftrightarrow (p\ (ap\ (ap \\
& \quad (c.2EquantHeuristics.2EGUESS_FORALL_GAP\ A.27a\ A.27b)\ V0i) \\
& \quad (\lambda V7x \in A.27b.(ap\ V1P\ V7x)))) \wedge ((p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_FORALL_GAP \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V8x \in A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V8x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_EXISTS_GAP\ A.27a\ A.27b) \\
& \quad V0i)\ (\lambda V9x \in A.27b.(ap\ V1P\ V9x)))) \wedge ((p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_EXISTS_POIN \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V10x \in A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V10x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_FORALL_POINT\ A.27a \\
& \quad A.27b)\ V0i)\ (\lambda V11x \in A.27b.(ap\ V1P\ V11x)))) \wedge ((p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_FORALL_ \\
& \quad A.27a\ A.27b)\ V0i)\ (\lambda V12x \in A.27b.(ap\ c.2Ebool.2E.7E\ (ap\ V1P\ V12x)))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c.2EquantHeuristics.2EGUESS_EXISTS_POINT\ A.27a \\
& \quad A.27b)\ V0i)\ (\lambda V13x \in A.27b.(ap\ V1P\ V13x)))))))))
\end{aligned}$$

(7)

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\
& \quad \forall V0i \in (A_{.27b}^{A_{.27a}}).(\forall V1P \in (2^{A_{.27b}}).(\forall V2Q \in \\
& \quad (2^{A_{.27b}}).(\forall V3iK \in A_{.27b}.(\forall V4q \in 2.(\forall V5p \in \\
& \quad 2.(((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL_POINT \\
A_{.27a} A_{.27b}) V0i) (\lambda V6x \in A_{.27b}.(ap V1P V6x)))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXIST \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V7x \in A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E \\
& \quad (ap V1P V7x)) (ap V2Q V7x)))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_POIN \\
A_{.27a} A_{.27b}) V0i) (\lambda V8x \in A_{.27b}.(ap V2Q V8x)))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXIST \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V9x \in A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E \\
& \quad (ap V1P V9x)) (ap V2Q V9x)))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V10x \in A_{.27b}.(ap V1P V10x)))) \wedge (p (ap (ap (\\
& \quad c_2EquantHeuristics_2EGUESS_EXISTS A_{.27a} A_{.27b}) V0i) (\lambda V11x \in \\
& \quad A_{.27b}.(ap V2Q V11x)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V12x \in A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E \\
& \quad (ap V1P V12x)) (ap V2Q V12x)))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL_G \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V13x \in A_{.27b}.(ap V1P V13x)))) \wedge (p (ap (ap (\\
& \quad c_2EquantHeuristics_2EGUESS_EXISTS_GAP A_{.27a} A_{.27b}) V0i) \\
& \quad (\lambda V14x \in A_{.27b}.(ap V2Q V14x)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V15x \in A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E \\
& \quad (ap V1P V15x)) (ap V2Q V15x)))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS \\
& \quad ty_2Eone_2Eone A_{.27b}) (\lambda V16xxx \in ty_2Eone_2Eone.V3iK)) (\lambda V17x \in \\
& \quad A_{.27b}.(ap V1P V17x)))) \wedge (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad ty_2Eone_2Eone A_{.27b}) (\lambda V18xxx \in ty_2Eone_2Eone.V3iK)) (\lambda V19x \in \\
& \quad A_{.27b}.(ap V2Q V19x)))))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad ty_2Eone_2Eone A_{.27b}) (\lambda V20xxx \in ty_2Eone_2Eone.V3iK)) (\lambda V21x \in \\
& \quad A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E (ap V1P V21x)) (ap V2Q V21x)))))) \wedge \\
& \quad (((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS A_{.27a} A_{.27b}) \\
& \quad V0i) (\lambda V22x \in A_{.27b}.(ap V1P V22x)))) \Rightarrow (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V23x \in A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E \\
& \quad (ap V1P V23x)) V4q)))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V24x \in A_{.27b}.(ap V2Q V24x)))) \Rightarrow (p (ap (ap (\\
& \quad c_2EquantHeuristics_2EGUESS_FORALL A_{.27a} A_{.27b}) V0i) (\lambda V25x \in \\
& \quad A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E V5p) (ap V2Q V25x)))))) \wedge (((\\
& \quad (p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_POINT A_{.27a} \\
& \quad A_{.27b}) V0i) (\lambda V26x \in A_{.27b}.(ap V1P V26x)))) \wedge (p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V27x \in A_{.27b}.(ap V2Q V27x)))))) \Rightarrow (p (ap (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_FORALL_POINT A_{.27a} A_{.27b}) V0i) \\
& \quad (\lambda V28x \in A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E (ap V1P V28x)) (ap \\
& \quad V2Q V28x)))))) \wedge (((p (ap (ap (c_2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V29x \in A_{.27b}.(ap V1P V29x)))) \Rightarrow (p (ap (ap (\\
& \quad c_2EquantHeuristics_2EGUESS_FORALL_GAP A_{.27a} A_{.27b}) V0i) \\
& \quad (\lambda V30x \in A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E (ap V1P V30x)) (ap \\
& \quad V2Q V30x)))))) \wedge ((p (ap (ap (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_{.27a} A_{.27b}) V0i) (\lambda V31x \in A_{.27b}.(ap V2Q V31x)))) \Rightarrow (p (ap (ap (\\
& \quad c_2EquantHeuristics_2EGUESS_FORALL_GAP A_{.27a} A_{.27b}) V0i) \\
& \quad (\lambda V32x \in A_{.27b}.(ap (ap c_2Emin_2E_3D_3D_3E (ap V1P V32x)) (ap \\
& \quad V2Q V32x)))))))))
\end{aligned}$$