

thm_2EquantHeuristics_2EGUESS_RULES_NEG
 (TMUFMMWTmgKL-
 CLb9waS1h16y2aSBSmMcrMG)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2EF}))$

Definition 7 We define `c_2EquantHeuristics_2EGUESS_FORALL_POINT` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27a) (\lambda V2fv \in A_27a.$

Definition 8 We define `c_2EquantHeuristics_2EGUESS_EXISTS_POINT` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27a) (\lambda V2fv \in A_27a.$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } P))))$

Definition 11 We define `c_2EquantHeuristics_2EGUESS_FORALL_GAP` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27b) (\lambda V2v \in A_27b. (\text{ap } P v))))$

Definition 12 We define `c_2EquantHeuristics_2EGUESS_EXISTS_GAP` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27b) (\lambda V2v \in A_27b. (\text{ap } P v))))$

Definition 13 We define `c_2EquantHeuristics_2EGUESS_FORALL` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0i \in (A_27b^{A_27a}). \lambda V1P \in (2^{A_27b}). (\text{ap } (\text{c_2Ebool_2E_21 } A_27b) (\lambda V2v \in A_27b. (\text{ap } P v))))$

Definition 14 We define `c_2EquantHeuristics_2EGUESS_EXISTS` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27$

Definition 15 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in \\ & A_{.27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ (\lambda V2x \in \\
& \quad A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V2x)))))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V3x \in A_27b.(ap\ V1P\ V3x)))))) \wedge (((p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_FORALL\ A_27a\ A_27b)\ V0i)\ (\lambda V4x \in \\
& \quad A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V4x)))))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V5x \in A_27b.(ap\ V1P\ V5x)))))) \wedge (((p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b)\ V0i) \\
& \quad (\lambda V6x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V6x)))))) \Leftrightarrow (p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_FORALL_GAP\ A_27a\ A_27b)\ V0i) \\
& \quad (\lambda V7x \in A_27b.(ap\ V1P\ V7x)))))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V8x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V8x)))))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b) \\
& \quad V0i)\ (\lambda V9x \in A_27b.(ap\ V1P\ V9x)))))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POIN \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V10x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V10x)))))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27a \\
& \quad A_27b)\ V0i)\ (\lambda V11x \in A_27b.(ap\ V1P\ V11x)))))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_ \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V12x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V12x)))))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT\ A_27a \\
& \quad A_27b)\ V0i)\ (\lambda V13x \in A_27b.(ap\ V1P\ V13x))))))))) \\
& \hspace{10em} (8)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ (\lambda V2x \in \\
& \quad A_27b.(ap\ V1P\ V2x)))))) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V3x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V3x)))))) \wedge \\
& \quad (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a \\
& \quad A_27b)\ V0i)\ (\lambda V4x \in A_27b.(ap\ V1P\ V4x)))))) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_G \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V5x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V5x)))))) \wedge \\
& \quad (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT\ A_27a \\
& \quad A_27b)\ V0i)\ (\lambda V6x \in A_27b.(ap\ V1P\ V6x)))))) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_PO \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V7x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V7x)))))) \wedge \\
& \quad (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL\ A_27a\ A_27b) \\
& \quad V0i)\ (\lambda V8x \in A_27b.(ap\ V1P\ V8x)))))) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V9x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V9x)))))) \wedge \\
& \quad (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP\ A_27a \\
& \quad A_27b)\ V0i)\ (\lambda V10x \in A_27b.(ap\ V1P\ V10x)))))) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_ \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V11x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V11x)))))) \wedge \\
& \quad (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27a \\
& \quad A_27b)\ V0i)\ (\lambda V12x \in A_27b.(ap\ V1P\ V12x)))))) \Rightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_P \\
& \quad A_27a\ A_27b)\ V0i)\ (\lambda V13x \in A_27b.(ap\ c_2Ebool_2E_7E\ (ap\ V1P\ V13x)))))))))
\end{aligned}$$