

thm_2EquantHeuristics_2EISR_exists (TM- creYMPEVkraxB9AbnWvGzozwk4RZCrMNE)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A P)))$

Definition 8 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \tag{1}$$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let `c_2Esum_2EABS_sum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Esum_2EABS_sum A.27a A.27b \in ((ty_2Esum_2Esum A.27a A.27b)^{((2^{A-27b})^{A-27a})^2}) \tag{2}$$

Definition 10 We define `c_2Esum_2EINL` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0e \in A.27a.(ap (c_2Esum_2EABS_sum A.27a A.27b) V0e)$

Definition 11 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_5C_2F))$

Definition 12 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$

Let $c_2Esum_2EISR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISR \\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (3)$$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (5)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in \\ A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ p\ V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in A_27a. (\exists V1x \in \\ A_27a. (V1x = V0a))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0y \in A_27a. (\forall V1x \in A_27a. (((ap (c_2Esum_2EINL \\ A_27a\ A_27b)\ V1x) = (ap (c_2Esum_2EINL\ A_27a\ A_27b)\ V0y)) \Leftrightarrow (V1x = \\ V0y)))) \wedge (\forall V2y \in A_27b. (\forall V3x \in A_27b. (((ap (c_2Esum_2EINR \\ A_27a\ A_27b)\ V3x) = (ap (c_2Esum_2EINR\ A_27a\ A_27b)\ V2y)) \Leftrightarrow (V3x = \\ V2y)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \forall V0ss \in (ty_2Esum_2Esum\ A_{.27a}\ A_{.27b}).((\exists V1x \in A_{.27a}. \\ & (V0ss = (ap\ (c_2Esum_2EINL\ A_{.27a}\ A_{.27b})\ V1x))) \vee (\exists V2y \in A_{.27b}. \\ & (V0ss = (ap\ (c_2Esum_2EINR\ A_{.27a}\ A_{.27b})\ V2y)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\neg((ap\ (c_2Esum_2EINL \\ & A_{.27a}\ A_{.27b})\ V0x) = (ap\ (c_2Esum_2EINR\ A_{.27a}\ A_{.27b})\ V1y)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & (\forall V0x \in A_{.27b}.(p\ (ap\ (c_2Esum_2EISR\ A_{.27a}\ A_{.27b})\ (ap\ (c_2Esum_2EINR \\ & A_{.27a}\ A_{.27b})\ V0x)))) \wedge (\forall V1y \in A_{.27a}.(\neg(p\ (ap\ (c_2Esum_2EISR \\ & A_{.27a}\ A_{.27b})\ (ap\ (c_2Esum_2EINL\ A_{.27a}\ A_{.27b})\ V1y)))))) \end{aligned} \quad (15)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \forall V0x \in (ty_2Esum_2Esum\ A_{.27a}\ A_{.27b}).((p\ (ap\ (c_2Esum_2EISR \\ & A_{.27a}\ A_{.27b})\ V0x)) \Leftrightarrow (\exists V1r \in A_{.27b}.(V0x = (ap\ (c_2Esum_2EINR \\ & A_{.27a}\ A_{.27b})\ V1r)))))) \end{aligned}$$