

thm\_2EquantHeuristics\_2ELIST\_LENGTH\_1  
(TMKm9WSAFZYEAnDuuX1kVP8AHfvTuvQE2Wf)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow p \Rightarrow Q)$  of type  $\iota$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (2)$$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a})))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_2F))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 8** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then}$  (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

**Definition 9** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 10** We define  $c\_2Ebool\_2E2F5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in$

**Definition 11** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

**Definition 12** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 13** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Earithmetic\_2E2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Earithmetic\_2E2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 14** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (14)$$

**Definition 15** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 16** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 17** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 18** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 19** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 20** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (15)$$

**Definition 21** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 22** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic$

**Definition 23** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum)^{(ty\_2Elist\_2Elist\ A\_27a)} \quad (16)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \quad (17)$$

Assume the following.

$$(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ c\_2Enum\_2ESUC\ V0m))\ V1n)))) \quad (18)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0c)\ V0c) = c\_2Enum\_2E0)) \quad (19)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (p (ap (ap (c\_2Earithmetic\_2E\_3E\_3D V0n) V1m)) \Leftrightarrow (p (ap (ap (c\_2Earithmetic\_2E\_3C\_3D V1m) V0n)))))) \tag{20}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in 2. ((\exists V2x \in A\_27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a. (p (ap V0P V3x)) \wedge (p V1Q)))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist A\_27a). ((ap (ap (c\_2Elist\_2EAPPEND A\_27a) (c\_2Elist\_2ENIL A\_27a)) V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist A\_27a). (\forall V2l2 \in (ty\_2Elist\_2Elist A\_27a). (\forall V3h \in A\_27a. ((ap (ap (c\_2Elist\_2EAPPEND A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V3h) V1l1)) V2l2) = (ap (ap (c\_2Elist\_2ECONS A\_27a) V3h) (ap (ap (c\_2Elist\_2EAPPEND A\_27a) V1l1) V2l2)))))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist A\_27a). (((ap (c\_2Elist\_2ELENGTH A\_27a) V0l) = c\_2Enum\_2E0) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL A\_27a)))) \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\
& A.27a).(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V0l) = c\_2Enum\_2E0) \Leftrightarrow ( \\
& V0l = (c\_2Elist\_2ENIL\ A.27a)))) \wedge ((\forall V1l \in (ty\_2Elist\_2Elist \\
& A.27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH \\
& A.27a)\ V1l) = (ap\ c\_2Enum\_2ESUC\ V2n)) \Leftrightarrow (\exists V3h \in A.27a.(\exists V4l.27 \in \\
& (ty\_2Elist\_2Elist\ A.27a).(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V4l.27) = \\
& V2n) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A.27a)\ V3h)\ V4l.27)))))) \wedge \\
& (\forall V5l \in (ty\_2Elist\_2Elist\ A.27a).(\forall V6n1 \in ty\_2Enum\_2Enum. \\
& (\forall V7n2 \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A.27a) \\
& V5l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V6n1)\ V7n2)) \Leftrightarrow (\exists V8l1 \in \\
& (ty\_2Elist\_2Elist\ A.27a).(\exists V9l2 \in (ty\_2Elist\_2Elist\ A.27a). \\
& (((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V8l1) = V6n1) \wedge ((ap\ (c\_2Elist\_2ELENGTH \\
& A.27a)\ V9l2) = V7n2) \wedge (V5l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A.27a)\ V8l1) \\
& V9l2))))))))))
\end{aligned}$$

(27)

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\
& A.27a).(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V0l) = c\_2Enum\_2E0) \Leftrightarrow ( \\
& V0l = (c\_2Elist\_2ENIL\ A.27a)))) \wedge ((\forall V1l \in (ty\_2Elist\_2Elist \\
& A.27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH \\
& A.27a)\ V1l) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V2n))) \Leftrightarrow (\exists V3h \in A.27a.(\exists V4l.27 \in (ty\_2Elist\_2Elist \\
& A.27a).(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V4l.27) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V2n)))) \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \wedge \\
& (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A.27a)\ V3h)\ V4l.27)))))) \wedge ((\forall V5l \in \\
& (ty\_2Elist\_2Elist\ A.27a).(\forall V6n \in ty\_2Enum\_2Enum.(((ap \\
& (c\_2Elist\_2ELENGTH\ A.27a)\ V5l) = (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V6n))) \Leftrightarrow (\exists V7h \in A.27a.(\exists V8l.27 \in \\
& (ty\_2Elist\_2Elist\ A.27a).(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V8l.27) = \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V6n)))) \wedge \\
& (V5l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A.27a)\ V7h)\ V8l.27)))))) \wedge (\forall V9l \in \\
& (ty\_2Elist\_2Elist\ A.27a).(\forall V10n1 \in ty\_2Enum\_2Enum.(\forall V11n2 \in \\
& ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V9l) = (ap\ (ap \\
& c\_2Earithmetic\_2E\_2B\ V10n1)\ V11n2)) \Leftrightarrow (\exists V12l1 \in (ty\_2Elist\_2Elist \\
& A.27a).(\exists V13l2 \in (ty\_2Elist\_2Elist\ A.27a).(((ap\ (c\_2Elist\_2ELENGTH \\
& A.27a)\ V12l1) = V10n1) \wedge ((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V13l2) = \\
& V11n2) \wedge (V9l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A.27a)\ V12l1)\ V13l2))))))))))
\end{aligned}$$

(28)

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.( \\
& \quad \forall V1m \in ty\_2Enum\_2Enum. (\forall V2l \in (ty\_2Elist\_2Elist \\
& \quad A_{.27a}). ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad V0n)\ V1m))\ (ap\ (c\_2Elist\_2ELENGTH\ A_{.27a}\ V2l))) \Leftrightarrow (\exists V3l1 \in \\
& \quad (ty\_2Elist\_2Elist\ A_{.27a}). (\exists V4l2 \in (ty\_2Elist\_2Elist\ A_{.27a}). \\
& \quad (((ap\ (c\_2Elist\_2ELENGTH\ A_{.27a}\ V3l1) = V0n) \wedge ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V1m)\ (ap\ (c\_2Elist\_2ELENGTH\ A_{.27a}\ V4l2)))) \wedge (V2l = (ap\ (ap\ (c\_2Elist\_2EAPPEND \\
& \quad A_{.27a}\ V3l1)\ V4l2))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.( \\
& \quad \forall V1l \in (ty\_2Elist\_2Elist\ A_{.27a}). ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ A_{.27a}\ V1l))) \Leftrightarrow (\exists V2l1 \in (ty\_2Elist\_2Elist \\
& \quad A_{.27a}). (\exists V3l2 \in (ty\_2Elist\_2Elist\ A_{.27a}). (((ap\ (c\_2Elist\_2ELENGTH \\
& \quad A_{.27a}\ V2l1) = V0n) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A_{.27a}\ V2l1) \\
& \quad V3l2))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
& \quad A_{.27a}). ((c\_2Enum\_2E0 = (ap\ (c\_2Elist\_2ELENGTH\ A_{.27a}\ V0l)) \Leftrightarrow ( \\
& \quad \quad V0l = (c\_2Elist\_2ENIL\ A_{.27a}))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
& \quad A_{.27a}). (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ (c\_2Elist\_2ELENGTH \\
& \quad A_{.27a}\ V0l))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A_{.27a}))) \wedge \\
& \quad (((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))\ (ap\ (c\_2Elist\_2ELENGTH \\
& \quad A_{.27a}\ V0l))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A_{.27a}))) \wedge ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\_3D \\
& \quad c\_2Enum\_2E0)\ (ap\ (c\_2Elist\_2ELENGTH\ A_{.27a}\ V0l))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL \\
& \quad A_{.27a}))) \wedge ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ (c\_2Elist\_2ELENGTH \\
& \quad A_{.27a}\ V0l))\ c\_2Enum\_2E0)) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A_{.27a}))))))
\end{aligned} \tag{33}$$

**Theorem 1**

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
& A_{.27a}). (\forall V1x \in ty\_2Enum\_2Enum. (((ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V0l) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) \Leftrightarrow (\exists V2e1 \in A_{.27a}. (V0l = (ap (ap \\
& (c\_2Elist\_2ECONS A_{.27a}) V2e1) (c\_2Elist\_2ENIL A_{.27a})))))) \wedge (( \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)) = \\
& (ap (c\_2Elist\_2ELENGTH A_{.27a}) V0l)) \Leftrightarrow (\exists V3e1 \in A_{.27a}. (V0l = \\
& (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V3e1) (c\_2Elist\_2ENIL A_{.27a})))))) \wedge \\
& (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V0l))) \Leftrightarrow (\exists V4l_{.27} \in (ty\_2Elist\_2Elist A_{.27a}). (\exists V5e1 \in \\
& A_{.27a}. (V0l = (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V5e1) V4l_{.27})))))) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3E (ap (c\_2Elist\_2ELENGTH A_{.27a}) \\
& V0l)) c\_2Enum\_2E0)) \Leftrightarrow (\exists V6l_{.27} \in (ty\_2Elist\_2Elist A_{.27a}). \\
& (\exists V7e1 \in A_{.27a}. (V0l = (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V7e1) \\
& V6l_{.27})))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) (ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V0l)) \Leftrightarrow (\exists V8l_{.27} \in (ty\_2Elist\_2Elist A_{.27a}). (\exists V9e1 \in \\
& A_{.27a}. (V0l = (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V9e1) V8l_{.27})))))) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3E\_3D (ap (c\_2Elist\_2ELENGTH A_{.27a}) \\
& V0l)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO)))) \Leftrightarrow (\exists V10l_{.27} \in (ty\_2Elist\_2Elist \\
& A_{.27a}). (\exists V11e1 \in A_{.27a}. (V0l = (ap (ap (c\_2Elist\_2ECONS A_{.27a}) \\
& V11e1) V10l_{.27})))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) V1x)) (ap (c\_2Elist\_2ELENGTH A_{.27a}) \\
& V0l)) \Leftrightarrow (\exists V12l_{.27} \in (ty\_2Elist\_2Elist A_{.27a}). (\exists V13e1 \in \\
& A_{.27a}. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1x) (ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V12l_{.27}))) \wedge (V0l = (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V13e1) \\
& V12l_{.27})))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3E\_3D (ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V0l)) (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V1x))) \Leftrightarrow \\
& (\exists V14l_{.27} \in (ty\_2Elist\_2Elist A_{.27a}). (\exists V15e1 \in A_{.27a}. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1x) (ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V14l_{.27}))) \wedge (V0l = (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V15e1) \\
& V14l_{.27})))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1x) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO)))) (ap (c\_2Elist\_2ELENGTH A_{.27a}) V0l)) \Leftrightarrow \\
& (\exists V16l_{.27} \in (ty\_2Elist\_2Elist A_{.27a}). (\exists V17e1 \in A_{.27a}. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1x) (ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V16l_{.27}))) \wedge (V0l = (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V17e1) \\
& V16l_{.27})))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3E\_3D (ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V0l)) (ap (ap c\_2Earithmetic\_2E\_2B V1x) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \Leftrightarrow (\exists V18l_{.27} \in \\
& (ty\_2Elist\_2Elist A_{.27a}). (\exists V19e1 \in A_{.27a}. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1x) (ap (c\_2Elist\_2ELENGTH A_{.27a}) V18l_{.27}))) \wedge (V0l = (ap (ap (c\_2Elist\_2ECONS \\
& A_{.27a}) V19e1) V18l_{.27})))))) \wedge (((ap (c\_2Elist\_2ELENGTH A_{.27a}) \\
& V0l) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V1x)) \Leftrightarrow ( \\
& \exists V20l_{.27} \in (ty\_2Elist\_2Elist A_{.27a}). (\exists V21e1 \in A_{.27a}. \\
& (((ap (c\_2Elist\_2ELENGTH A_{.27a}) V20l_{.27}) = V1x) \wedge (V0l = (ap (ap ( \\
& c\_2Elist\_2ECONS A_{.27a}) V21e1) V20l_{.27})))))) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
& V1x) = (ap (c\_2Elist\_2ELENGTH A_{.27a}) V0l)) \Leftrightarrow (\exists V22l_{.27} \in ( \\
& ty\_2Elist\_2Elist A_{.27a}). (\exists V23e1 \in A_{.27a}. (((ap (c\_2Elist\_2ELENGTH \\
& A_{.27a}) V22l_{.27}) = V1x) \wedge (V0l = (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V23e1) \\
& V22l_{.27})))))) \wedge (((ap (c\_2Elist\_2ELENGTH A_{.27a}) V0l) = (ap (ap \\
& c\_2Earithmetic\_2E\_2B V1x) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO)))) \Leftrightarrow (\exists V24l_{.27} \in (ty\_2Elist\_2Elist \\
& A_{.27a}). (\exists V25e1 \in A_{.27a}. (((ap (c\_2Elist\_2ELENGTH A_{.27a}) \\
\end{aligned}$$