

thm_2EquantHeuristics_2ELIST_LENGTH_2
(TMJwtKnw2ZSJUh465uLpsWLX5PWBQwdEqJm)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty_2Elist_2Elist \ A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Elist_2ECONS \ A_27a \in (((ty_2Elist_2Elist \ A_27a)^{(ty_2Elist_2Elist \ A_27a)})^{A_27a}) \quad (2)$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap \ (ap \ (c_2Emin_2E_3D \ (2^{A_27a})) \ (\lambda V1x \in 2.V1x)) \ (\lambda V1x \in 2.V1x)))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap \ (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap \ (ap \ c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2E_2F))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \quad (3)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 8 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define c_2Ebool_2E2F5C to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Definition 11 We define c_2Ebool_2ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Definition 12 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 13 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Earithmetic_2E2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Earithmetic_2E2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 14 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)} \quad (14)$$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 18 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 19 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (15)$$

Definition 21 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 22 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Definition 23 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist\ A_27a)} \quad (16)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V0m)))) \quad (17)$$

Assume the following.

$$(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ c_2Enum_2ESUC\ V0m))\ V1n)))) \quad (18)$$

Assume the following.

$$(\forall V0c \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2D\ V0c)\ V0c) = c_2Enum_2E0)) \quad (19)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (p (ap (ap (c_2Earithmetic_2E_3E_3D V0n) V1m)) \Leftrightarrow (p (ap (ap (c_2Earithmetic_2E_3C_3D V1m) V0n)))))) \tag{20}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{21}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{22}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\exists V2x \in A_27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p (ap V0P V3x)) \wedge (p V1Q)))))) \tag{23}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \tag{24}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist A_27a). ((ap (ap (c_2Elist_2EAPPEND A_27a) (c_2Elist_2ENIL A_27a)) V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_27a). (\forall V2l2 \in (ty_2Elist_2Elist A_27a). (\forall V3h \in A_27a. ((ap (ap (c_2Elist_2EAPPEND A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V3h) V1l1)) V2l2) = (ap (ap (c_2Elist_2ECONS A_27a) V3h) (ap (ap (c_2Elist_2EAPPEND A_27a) V1l1) V2l2)))))))))) \tag{25}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist A_27a). (((ap (c_2Elist_2ELENGTH A_27a) V0l) = c_2Enum_2E0) \Leftrightarrow (V0l = (c_2Elist_2ENIL A_27a)))) \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A.27a).(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V0l) = c_2Enum_2E0) \Leftrightarrow (\\
& V0l = (c_2Elist_2ENIL\ A.27a)))) \wedge ((\forall V1l \in (ty_2Elist_2Elist \\
& A.27a).(\forall V2n \in ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH \\
& A.27a)\ V1l) = (ap\ c_2Enum_2ESUC\ V2n)) \Leftrightarrow (\exists V3h \in A.27a.(\exists V4l.27 \in \\
& (ty_2Elist_2Elist\ A.27a).(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V4l.27) = \\
& V2n) \wedge (V1l = (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V3h)\ V4l.27)))))) \wedge \\
& (\forall V5l \in (ty_2Elist_2Elist\ A.27a).(\forall V6n1 \in ty_2Enum_2Enum. \\
& (\forall V7n2 \in ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH\ A.27a) \\
& V5l) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V6n1)\ V7n2)) \Leftrightarrow (\exists V8l1 \in \\
& (ty_2Elist_2Elist\ A.27a).(\exists V9l2 \in (ty_2Elist_2Elist\ A.27a). \\
& (((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V8l1) = V6n1) \wedge ((ap\ (c_2Elist_2ELENGTH \\
& A.27a)\ V9l2) = V7n2) \wedge (V5l = (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V8l1) \\
& V9l2))))))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A.27a).(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V0l) = c_2Enum_2E0) \Leftrightarrow (\\
& V0l = (c_2Elist_2ENIL\ A.27a)))) \wedge ((\forall V1l \in (ty_2Elist_2Elist \\
& A.27a).(\forall V2n \in ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH \\
& A.27a)\ V1l) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& V2n))) \Leftrightarrow (\exists V3h \in A.27a.(\exists V4l.27 \in (ty_2Elist_2Elist \\
& A.27a).(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V4l.27) = (ap\ (ap\ c_2Earithmetic_2E_2D \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V2n)))) \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \wedge \\
& (V1l = (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V3h)\ V4l.27)))))) \wedge ((\forall V5l \in \\
& (ty_2Elist_2Elist\ A.27a).(\forall V6n \in ty_2Enum_2Enum.(((ap \\
& (c_2Elist_2ELENGTH\ A.27a)\ V5l) = (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ V6n))) \Leftrightarrow (\exists V7h \in A.27a.(\exists V8l.27 \in \\
& (ty_2Elist_2Elist\ A.27a).(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V8l.27) = \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V6n))) \wedge \\
& (V5l = (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V7h)\ V8l.27)))))) \wedge ((\forall V9l \in \\
& (ty_2Elist_2Elist\ A.27a).(\forall V10n1 \in ty_2Enum_2Enum.(\forall V11n2 \in \\
& ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V9l) = (ap\ (ap \\
& c_2Earithmetic_2E_2B\ V10n1)\ V11n2)) \Leftrightarrow (\exists V12l1 \in (ty_2Elist_2Elist \\
& A.27a).(\exists V13l2 \in (ty_2Elist_2Elist\ A.27a).(((ap\ (c_2Elist_2ELENGTH \\
& A.27a)\ V12l1) = V10n1) \wedge ((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V13l2) = \\
& V11n2) \wedge (V9l = (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V12l1)\ V13l2))))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
 &(((ap\ c_2Enum_2ESUC\ c_2Earithmetic_2EZERO) = (ap\ c_2Earithmetic_2EBIT1 \\
 &\quad c_2Earithmetic_2EZERO)) \wedge (\forall V0n \in ty_2Enum_2Enum. ((ap \\
 &\quad c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2 \\
 &\quad V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT2 \\
 &\quad V1n)) = (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enum_2ESUC\ V1n))))))
 \end{aligned}
 \tag{29}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \quad \forall V1m \in ty_2Enum_2Enum. (\forall V2l \in (ty_2Elist_2Elist \\
& \quad A_{.27a}). ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& \quad V0n)\ V1m))\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a}\ V2l))) \Leftrightarrow (\exists V3l1 \in \\
& \quad (ty_2Elist_2Elist\ A_{.27a}). (\exists V4l2 \in (ty_2Elist_2Elist\ A_{.27a}). \\
& \quad (((ap\ (c_2Elist_2ELENGTH\ A_{.27a}\ V3l1) = V0n) \wedge ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& \quad V1m)\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a}\ V4l2)))) \wedge (V2l = (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_{.27a}\ V3l1)\ V4l2))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \quad \forall V1l \in (ty_2Elist_2Elist\ A_{.27a}). ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& \quad V0n)\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a}\ V1l))) \Leftrightarrow (\exists V2l1 \in (ty_2Elist_2Elist \\
& \quad A_{.27a}). (\exists V3l2 \in (ty_2Elist_2Elist\ A_{.27a}). (((ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a}\ V2l1) = V0n) \wedge (V1l = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_{.27a}\ V2l1) \\
& \quad V3l2))))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& \quad A_{.27a}). ((c_2Enum_2E0 = (ap\ (c_2Elist_2ELENGTH\ A_{.27a}\ V0l)) \Leftrightarrow (\\
& \quad \quad V0l = (c_2Elist_2ENIL\ A_{.27a}))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& \quad A_{.27a}). (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a}\ V0l))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_{.27a})) \wedge \\
& \quad (((p\ (ap\ (ap\ c_2Earithmetic_2E_3E\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ (ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a}\ V0l))) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_{.27a})) \wedge ((p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D \\
& \quad c_2Enum_2E0)\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a}\ V0l))) \Leftrightarrow (V0l = (c_2Elist_2ENIL \\
& \quad A_{.27a}))) \wedge ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a}\ V0l))\ c_2Enum_2E0)) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_{.27a}))))))
\end{aligned} \tag{34}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& \quad A_{.27a}).(\forall V1x \in ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a})\ V0l) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2 \\
& \quad c_2Earithmetic_2EZERO))) \Leftrightarrow (\exists V2e1 \in A_{.27a}.(\exists V3e2 \in \\
& \quad A_{.27a}.(V0l = (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V2e1)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_{.27a})\ V3e2)\ (c_2Elist_2ENIL\ A_{.27a})))))) \wedge (((ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)) = (ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a})\ V0l)) \Leftrightarrow (\exists V4e1 \in A_{.27a}.(\exists V5e2 \in A_{.27a}.(V0l = \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V4e1)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_{.27a})\ V5e2)\ (c_2Elist_2ENIL\ A_{.27a})))))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) \\
& \quad (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V0l)) \Leftrightarrow (\exists V6l_27 \in (ty_2Elist_2Elist \\
& \quad A_{.27a}).(\exists V7e1 \in A_{.27a}.(\exists V8e2 \in A_{.27a}.(V0l = (ap\ (ap \\
& \quad (c_2Elist_2ECONS\ A_{.27a})\ V7e1)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a}) \\
& \quad V8e2)\ V6l_27)))))) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3E\ (ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a})\ V0l))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \Leftrightarrow (\exists V9l_27 \in (ty_2Elist_2Elist \\
& \quad A_{.27a}).(\exists V10e1 \in A_{.27a}.(\exists V11e2 \in A_{.27a}.(V0l = (ap \\
& \quad (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V10e1)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a}) \\
& \quad V11e2)\ V9l_27)))))) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap \\
& \quad c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))) \\
& \quad (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V0l)) \Leftrightarrow (\exists V12l_27 \in (ty_2Elist_2Elist \\
& \quad A_{.27a}).(\exists V13e1 \in A_{.27a}.(\exists V14e2 \in A_{.27a}.(V0l = (ap \\
& \quad (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V13e1)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a}) \\
& \quad V14e2)\ V12l_27)))))) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D\ (\\
& \quad ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V0l))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))) \Leftrightarrow (\exists V15l_27 \in \\
& \quad (ty_2Elist_2Elist\ A_{.27a}).(\exists V16e1 \in A_{.27a}.(\exists V17e2 \in \\
& \quad A_{.27a}.(V0l = (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V16e1)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_{.27a})\ V17e2)\ V15l_27)))))) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\
& \quad c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ V1x))\ (ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a})\ V0l)) \Leftrightarrow (\exists V18l_27 \in (ty_2Elist_2Elist\ A_{.27a}).(\exists V19e1 \in \\
& \quad A_{.27a}.(\exists V20e2 \in A_{.27a}.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\
& \quad V1x)\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V18l_27))) \wedge (V0l = (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_{.27a})\ V19e1)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V20e2)\ V18l_27)))))) \wedge \\
& \quad (((p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a}) \\
& \quad V0l))\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ V1x)) \Leftrightarrow \\
& \quad (\exists V21l_27 \in (ty_2Elist_2Elist\ A_{.27a}).(\exists V22e1 \in A_{.27a}. \\
& \quad (\exists V23e2 \in A_{.27a}.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1x) \\
& \quad (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V21l_27))) \wedge (V0l = (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_{.27a})\ V22e1)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V23e2)\ V21l_27)))))) \wedge \\
& \quad (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& \quad V1x)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2 \\
& \quad c_2Earithmetic_2EZERO))))\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V0l)) \Leftrightarrow \\
& \quad (\exists V24l_27 \in (ty_2Elist_2Elist\ A_{.27a}).(\exists V25e1 \in A_{.27a}. \\
& \quad (\exists V26e2 \in A_{.27a}.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1x) \\
& \quad (ap\ (c_2Elist_2ELENGTH\ A_{.27a})\ V24l_27))) \wedge (V0l = (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_{.27a})\ V25e1)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V26e2)\ V24l_27)))))) \wedge \\
& \quad (((p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D\ (ap\ (c_2Elist_2ELENGTH\ A_{.27a}) \\
& \quad V0l))\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1x)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) \Leftrightarrow (\exists V27l_27 \in \\
& \quad (ty_2Elist_2Elist\ A_{.27a}).(\exists V28e1 \in A_{.27a}.(\exists V29e2 \in \\
& \quad A_{.27a}.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1x)\ (ap\ (c_2Elist_2ELENGTH \\
& \quad A_{.27a})\ V27l_27))) \wedge (V0l = (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V28e1) \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ A_{.27a})\ V29e2)\ V27l_27)))))) \wedge (((ap \\
& \quad (c_2Elist_2ELENGTH\ A_{.27a})\ V0l) = (ap\ (ap\ c_2Earithmetic_2E_2B \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))))
\end{aligned}$$