

# thm\_2EquantHeuristics\_2ELIST\_\_LENGTH\_\_3 (TMEsyRjooDqa8rpK9aSRrf5G1Mymrk5MjQR)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (2)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($

**Definition 6** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

Let  $c\_2Enum\_2EZZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZZERO\_REP \in \omega \tag{9}$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZZERO\_REP)$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 9** We define  $c\_2Ebool\_2E\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V2t \in 2.V2t))$

**Definition 10** We define  $c\_2Ebool\_2E\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

**Definition 11** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E\_2E21$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Let  $c\_2Earithmetic\_2E\_2EA : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2EA \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

**Definition 12** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 13** We define  $c\_2Ebool\_2E\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2E21$

Let  $c\_2Enumeral\_2EiSUB : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^2) \tag{12}$$

Let  $c\_2Earithmetic\_2E\_2ED : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2ED \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Let  $c\_2Earithmetic\_2E\_2EB : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2EB \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

**Definition 14** We define `c.Earithmic.EBIT2` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c.Earithmic.EBIT2 V0n) V0n)$

**Definition 15** We define `c.Enumeral.EiDUB` to be  $\lambda V0x \in ty\_2Enum\_2Enum.(ap (ap c.Earithmic.EBIT2 V0x) V0x)$

Let `c.Elist.EAPPEND` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c.Elist.EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (15)$$

**Definition 16** We define `c.Ebool.E3F` to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c.Emin.E40 A\_27a) V0P)))$

**Definition 17** We define `c.Eprim_rec.E3C` to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c.Earithmic.EBIT2 V0m) V1n)$

**Definition 18** We define `c.Ebool.E5C.2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c.Ebool.E21 2) (\lambda V2t \in 2.(ap (ap c.Earithmic.EBIT2 V0t1) V2t))))$

**Definition 19** We define `c.Earithmic.E3C.3D` to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c.Earithmic.EBIT2 V0m) V1n)$

**Definition 20** We define `c.Earithmic.E3E` to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c.Earithmic.EBIT2 V0m) V1n)$

**Definition 21** We define `c.Earithmic.E3E.3D` to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c.Earithmic.EBIT2 V0m) V1n)$

Let `c.Elist.ENIL` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c.Elist.ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (16)$$

**Definition 22** We define `c.Earithmic.EZERO` to be `c.Enum.E0`.

**Definition 23** We define `c.Earithmic.EBIT1` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c.Earithmic.EBIT2 V0n) V0n)$

**Definition 24** We define `c.Earithmic.ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let `c.Elist.ELENGTH` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c.Elist.ELENGTH A\_27a \in (ty\_2Enum\_2Enum)^{(ty\_2Elist\_2Elist A\_27a)} \quad (17)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(ap (ap (ap c.Earithmic.E2B V0m) V1n) = (ap (ap c.Earithmic.E2B V1n) V0m)))) \quad (18)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p (ap (ap (ap c.Eprim_rec.E3C V0m) V1n)) \Leftrightarrow (p (ap (ap c.Earithmic.E3C.3D (ap c.Enum.ESUC V0m)) V1n)))))) \quad (19)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2D V0c) V0c) = c\_2Enum\_2E0)) \quad (20)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmic\_2E\_3E\_3D V0n) V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D V1m) V0n)))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (22)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))) \quad (24)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in 2. ((\exists V2x \in A\_27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a. (p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (25)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \quad (26)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist A\_27a). ((ap (ap (c\_2Elist\_2EAPPEND A\_27a) (c\_2Elist\_2ENIL A\_27a)) V0l) = V0l) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist A\_27a). (\forall V2l2 \in (ty\_2Elist\_2Elist A\_27a). (\forall V3h \in A\_27a. ((ap (ap (c\_2Elist\_2EAPPEND A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V3h) V1l1)) V2l2) = (ap (ap (c\_2Elist\_2ECONS A\_27a) V3h) (ap (ap (c\_2Elist\_2EAPPEND A\_27a) V1l1) V2l2)))))))) \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = c\_2Enum\_2E0) \Leftrightarrow ( \\ V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = c\_2Enum\_2E0) \Leftrightarrow ( \\ V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\ A\_27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH \\ A\_27a)\ V1l) = (ap\ c\_2Enum\_2ESUC\ V2n)) \Leftrightarrow (\exists V3h \in A\_27a.(\exists V4l\_27 \in \\ ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l\_27) = \\ V2n) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V4l\_27)))))) \wedge \\ (\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6n1 \in ty\_2Enum\_2Enum. \\ (\forall V7n2 \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\ V5l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V6n1)\ V7n2)) \Leftrightarrow (\exists V8l1 \in \\ ty\_2Elist\_2Elist\ A\_27a).(\exists V9l2 \in (ty\_2Elist\_2Elist\ A\_27a). \\ (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V8l1) = V6n1) \wedge (((ap\ (c\_2Elist\_2ELENGTH \\ A\_27a)\ V9l2) = V7n2) \wedge (V5l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V8l1) \\ V9l2)))))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\
& A.27a).(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V0l) = c\_2Enum\_2E0) \Leftrightarrow ( \\
& V0l = (c\_2Elist\_2ENIL\ A.27a)))) \wedge ((\forall V1l \in (ty\_2Elist\_2Elist \\
& A.27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH \\
& A.27a)\ V1l) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V2n))) \Leftrightarrow (\exists V3h \in A.27a.(\exists V4l\_27 \in (ty\_2Elist\_2Elist \\
& A.27a).(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V4l\_27) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V2n))) \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \wedge \\
& (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A.27a)\ V3h)\ V4l\_27)))))) \wedge ((\forall V5l \in \\
& (ty\_2Elist\_2Elist\ A.27a).(\forall V6n \in ty\_2Enum\_2Enum.(((ap \\
& (c\_2Elist\_2ELENGTH\ A.27a)\ V5l) = (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V6n))) \Leftrightarrow (\exists V7h \in A.27a.(\exists V8l\_27 \in \\
& (ty\_2Elist\_2Elist\ A.27a).(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V8l\_27) = \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V6n))) \wedge \\
& (V5l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A.27a)\ V7h)\ V8l\_27)))))) \wedge (\forall V9l \in \\
& (ty\_2Elist\_2Elist\ A.27a).(\forall V10n1 \in ty\_2Enum\_2Enum.(\forall V11n2 \in \\
& ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V9l) = (ap\ (ap \\
& c\_2Earithmetic\_2E\_2B\ V10n1)\ V11n2))) \Leftrightarrow (\exists V12l1 \in (ty\_2Elist\_2Elist \\
& A.27a).(\exists V13l2 \in (ty\_2Elist\_2Elist\ A.27a).(((ap\ (c\_2Elist\_2ELENGTH \\
& A.27a)\ V12l1) = V10n1) \wedge (((ap\ (c\_2Elist\_2ELENGTH\ A.27a)\ V13l2) = \\
& V11n2) \wedge (V9l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A.27a)\ V12l1)\ V13l2))))))))))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c\_2Enum\_2ESUC\ c\_2Earithmetic\_2EZERO) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum.((ap \\
& c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V1n)) = (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enum\_2ESUC\ V1n)))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) (ap c\_2Earithmic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\
& (ap c\_2Earithmic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n)))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))))))))))
\end{aligned} \tag{33}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1b \in 2. (\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3m \in ty\_2Enum\_2Enum. (((ap (ap (ap c\_2Enumeral\_2EiSUB \\
& V1b) c\_2Earithmetic\_2EZERO) V0x) = c\_2Earithmetic\_2EZERO) \wedge ( \\
& ((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) c\_2Earithmetic\_2EZERO) = \\
V2n) \wedge (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) c\_2Earithmetic\_2EZERO) = (ap c\_2Enumeral\_2EiDUB V2n)) \wedge \\
(((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) c\_2Earithmetic\_2EZERO) = (ap c\_2Earithmetic\_2EBIT1 V2n)) \wedge \\
(((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge ((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))))))))))))))))) \\
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V0n) \\
V1m)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V1m) V0n)) (ap c\_2Earithmetic\_2ENUMERAL (ap (ap (ap c\_2Enumeral\_2EiSUB \\
c\_2Ebool\_2ET) V0n) V1m))) c\_2Enum\_2E0)))) \\
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) \wedge ((ap\ c\_2Enumeral\_2EiDUB\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO)))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (\forall V2l \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V2l))) \Leftrightarrow (\exists V3l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\exists V4l2 \in (ty\_2Elist\_2Elist\ A\_27a). (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V3l1) = V0n) \wedge ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1m)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l2)))) \wedge (V2l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V3l1)\ V4l2)))))))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Leftrightarrow (\exists V2l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\exists V3l2 \in (ty\_2Elist\_2Elist\ A\_27a). (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V2l1) = V0n) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2l1)\ V3l2)))))))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a). ((c\_2Enum\_2E0 = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a). (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a))) \wedge \\
& \quad (((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a))) \wedge (((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\_3D\ c\_2Enum\_2E0)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a))) \wedge ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))\ c\_2Enum\_2E0)) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))))) \\
& \hspace{15em} (40)
\end{aligned}$$



**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
& \quad A\_27a).(\forall V1x \in ty\_2Enum\_2Enum.((((ap (c\_2Elist\_2ELENGTH \\
& \quad A\_27a) V0l) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \Leftrightarrow (\exists V2e1 \in \\
& \quad A\_27a.(\exists V3e2 \in A\_27a.(\exists V4e3 \in A\_27a.(V0l = (ap (ap \\
& \quad (c\_2Elist\_2ECONS A\_27a) V2e1) (ap (ap (c\_2Elist\_2ECONS A\_27a) \\
& \quad V3e2) (ap (ap (c\_2Elist\_2ECONS A\_27a) V4e3) (c\_2Elist\_2ENIL A\_27a)))))))))) \wedge \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 ( \\
& \quad ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = (ap (c\_2Elist\_2ELENGTH \\
& \quad A\_27a) V0l)) \Leftrightarrow (\exists V5e1 \in A\_27a.(\exists V6e2 \in A\_27a.(\exists V7e3 \in \\
& \quad A\_27a.(V0l = (ap (ap (c\_2Elist\_2ECONS A\_27a) V5e1) (ap (ap (c\_2Elist\_2ECONS \\
& \quad A\_27a) V6e2) (ap (ap (c\_2Elist\_2ECONS A\_27a) V7e3) (c\_2Elist\_2ENIL \\
& \quad A\_27a)))))))))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap (c\_2Elist\_2ELENGTH \\
& \quad A\_27a) V0l)) \Leftrightarrow (\exists V8l\_27 \in (ty\_2Elist\_2Elist A\_27a).(\exists V9e1 \in \\
& \quad A\_27a.(\exists V10e2 \in A\_27a.(\exists V11e3 \in A\_27a.(V0l = (ap ( \\
& \quad ap (c\_2Elist\_2ECONS A\_27a) V9e1) (ap (ap (c\_2Elist\_2ECONS A\_27a) \\
& \quad V10e2) (ap (ap (c\_2Elist\_2ECONS A\_27a) V11e3) V8l\_27)))))))))) \wedge \\
& \quad (((p (ap (ap c\_2Earithmetic\_2E\_3E (ap (c\_2Elist\_2ELENGTH A\_27a) \\
& \quad V0l)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
& \quad c\_2Earithmetic\_2EZERO)))) \Leftrightarrow (\exists V12l\_27 \in (ty\_2Elist\_2Elist \\
& \quad A\_27a).(\exists V13e1 \in A\_27a.(\exists V14e2 \in A\_27a.(\exists V15e3 \in \\
& \quad A\_27a.(V0l = (ap (ap (c\_2Elist\_2ECONS A\_27a) V13e1) (ap (ap (c\_2Elist\_2ECONS \\
& \quad A\_27a) V14e2) (ap (ap (c\_2Elist\_2ECONS A\_27a) V15e3) V12l\_27)))))))))) \wedge \\
& \quad (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
& \quad (ap (c\_2Elist\_2ELENGTH A\_27a) V0l)) \Leftrightarrow (\exists V16l\_27 \in (ty\_2Elist\_2Elist \\
& \quad A\_27a).(\exists V17e1 \in A\_27a.(\exists V18e2 \in A\_27a.(\exists V19e3 \in \\
& \quad A\_27a.(V0l = (ap (ap (c\_2Elist\_2ECONS A\_27a) V17e1) (ap (ap (c\_2Elist\_2ECONS \\
& \quad A\_27a) V18e2) (ap (ap (c\_2Elist\_2ECONS A\_27a) V19e3) V16l\_27)))))))))) \wedge \\
& \quad (((p (ap (ap c\_2Earithmetic\_2E\_3E\_3D (ap (c\_2Elist\_2ELENGTH A\_27a) \\
& \quad V0l)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \Leftrightarrow (\exists V20l\_27 \in \\
& \quad (ty\_2Elist\_2Elist A\_27a).(\exists V21e1 \in A\_27a.(\exists V22e2 \in \\
& \quad A\_27a.(\exists V23e3 \in A\_27a.(V0l = (ap (ap (c\_2Elist\_2ECONS A\_27a) \\
& \quad V21e1) (ap (ap (c\_2Elist\_2ECONS A\_27a) V22e2) (ap (ap (c\_2Elist\_2ECONS \\
& \quad A\_27a) V23e3) V20l\_27)))))))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& \quad c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
& \quad V1x)) (ap (c\_2Elist\_2ELENGTH A\_27a) V0l)) \Leftrightarrow (\exists V24l\_27 \in \\
& \quad (ty\_2Elist\_2Elist A\_27a).(\exists V25e1 \in A\_27a.(\exists V26e2 \in \\
& \quad A\_27a.(\exists V27e3 \in A\_27a.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V1x) (ap (c\_2Elist\_2ELENGTH A\_27a) V24l\_27)) \wedge (V0l = (ap (ap (c\_2Elist\_2ECONS \\
& \quad A\_27a) V25e1) (ap (ap (c\_2Elist\_2ECONS A\_27a) V26e2) (ap (ap (c\_2Elist\_2ECONS \\
& \quad A\_27a) V27e3) V24l\_27)))))))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3E\_3D \\
& \quad (ap (c\_2Elist\_2ELENGTH A\_27a) V0l)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)))) V1x)) \Leftrightarrow (\exists V28l\_27 \in (ty\_2Elist\_2Elist \\
& \quad A\_27a).(\exists V29e1 \in A\_27a.(\exists V30e2 \in A\_27a.(\exists V31e3 \in \\
& \quad A\_27a.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1x) (ap (c\_2Elist\_2ELENGTH \\
& \quad A\_27a) V28l\_27)) \wedge (V0l = (ap (ap (c\_2Elist\_2ECONS A\_27a) V29e1) \\
& \quad (ap (ap (c\_2Elist\_2ECONS A\_27a) V30e2) (ap (ap (c\_2Elist\_2ECONS \\
& \quad A\_27a) V31e3) V28l\_27)))))))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V1x) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \\
& \quad (ap (c\_2Elist\_2ELENGTH A\_27a) V0l)) \Leftrightarrow (\exists V32l\_27 \in (ty\_2Elist\_2Elist \\
& \quad A\_27a).(\exists V33e1 \in A\_27a.(\exists V34e2 \in A\_27a.(\exists V35e3 \in \\
& \quad A\_27a.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1x) (ap (c\_2Elist\_2ELENGTH \\
& \quad A\_27a) V32l\_27)) \wedge (V0l = (ap (ap (c\_2Elist\_2ECONS A\_27a) V33e1)
\end{aligned}$$