

thm_2EquantHeuristics_2ELIST_LENGTH_4 (TMbbQghav4p3cVyRGPRJZz9s41ZbtoYhkS5)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Elist_2ECONS A.27a \in (((ty_2Elist_2Elist A.27a)^{(ty_2Elist_2Elist A.27a)})^{A.27a}) \quad (2)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (3)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_2E21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{9}$$

Definition 7 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t)))$

Definition 10 We define c_2Ebool_2ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Definition 11 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E_2E21$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{10}$$

Let $c_2Earithmetic_2E_2EA : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2EA \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{11}$$

Definition 12 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 13 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E21$

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2) \tag{12}$$

Let $c_2Earithmetic_2E_2ED : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2ED \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{13}$$

Let $c_2Earithmetic_2E_2EB : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2EB \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{14}$$

Definition 14 We define `c.Earithmic.EBIT2` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c.Earithmic.EBIT2 V0n))$

Definition 15 We define `c.Enumeral.EiDUB` to be $\lambda V0x \in ty_2Enum_2Enum.(ap (ap c.Earithmic.EBIT2 V0x))$

Let `c.Elist.EAPPEND` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c.Elist.EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (15)$$

Definition 16 We define `c.Ebool.E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c.Emin.E_40 A_27a) V0P))))$

Definition 17 We define `c.Eprim_rec.E_3C` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c.Earithmic.EBIT2 V0m) V1n))$

Definition 18 We define `c.Ebool.E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c.Ebool.E_21 2) (\lambda V2t \in 2.(ap (ap c.Earithmic.EBIT2 V0t1) V2t))))))$

Definition 19 We define `c.Earithmic.E_3C_3D` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c.Earithmic.EBIT2 V0m) V1n))$

Definition 20 We define `c.Earithmic.E_3E` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c.Earithmic.EBIT2 V0m) V1n))$

Definition 21 We define `c.Earithmic.E_3E_3D` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c.Earithmic.EBIT2 V0m) V1n))$

Let `c.Elist.ENIL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c.Elist.ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (16)$$

Definition 22 We define `c.Earithmic.EZERO` to be `c.Enum.E0`.

Definition 23 We define `c.Earithmic.EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c.Earithmic.EBIT2 V0n))$

Definition 24 We define `c.Earithmic.ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `c.Elist.ELENGTH` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c.Elist.ELENGTH A_27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist A_27a)} \quad (17)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap (ap (ap c.Earithmic.E_2B V0m) V1n) = (ap (ap c.Earithmic.E_2B V1n) V0m)))) \quad (18)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p (ap (ap (ap c.Eprim_rec.E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c.Earithmic.E_3C_3D (ap c.Enum.ESUC V0m)) V1n)))))) \quad (19)$$

Assume the following.

$$(\forall V0c \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2D V0c) V0c) = c_2Enum_2E0)) \quad (20)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmic_2E_3E_3D V0n) V1m)) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D V1m) V0n)))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (22)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \quad (24)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\exists V2x \in A_27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (25)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \quad (26)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist A_27a). ((ap (ap (c_2Elist_2EAPPEND A_27a) (c_2Elist_2ENIL A_27a) V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_27a). (\forall V2l2 \in (ty_2Elist_2Elist A_27a). (\forall V3h \in A_27a. ((ap (ap (c_2Elist_2EAPPEND A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V3h) V1l1)) V2l2) = (ap (ap (c_2Elist_2ECONS A_27a) V3h) (ap (ap (c_2Elist_2EAPPEND A_27a) V1l1) V2l2)))))))) \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_27a).(((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l) = c_2Enum_2E0) \Leftrightarrow (\\ V0l = (c_2Elist_2ENIL\ A_27a)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_27a).(((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l) = c_2Enum_2E0) \Leftrightarrow (\\ V0l = (c_2Elist_2ENIL\ A_27a)))) \wedge (\forall V1l \in (ty_2Elist_2Elist \\ A_27a).(\forall V2n \in ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH \\ A_27a)\ V1l) = (ap\ c_2Enum_2ESUC\ V2n)) \Leftrightarrow (\exists V3h \in A_27a.(\exists V4l_27 \in \\ ty_2Elist_2Elist\ A_27a).(((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V4l_27) = \\ V2n) \wedge (V1l = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ V4l_27)))))) \wedge \\ (\forall V5l \in (ty_2Elist_2Elist\ A_27a).(\forall V6n1 \in ty_2Enum_2Enum. \\ (\forall V7n2 \in ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH\ A_27a) \\ V5l) = (ap\ (ap\ c_2Earithmic_2E_2B\ V6n1)\ V7n2)) \Leftrightarrow (\exists V8l1 \in \\ ty_2Elist_2Elist\ A_27a).(\exists V9l2 \in (ty_2Elist_2Elist\ A_27a). \\ (((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V8l1) = V6n1) \wedge (((ap\ (c_2Elist_2ELENGTH \\ A_27a)\ V9l2) = V7n2) \wedge (V5l = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V8l1) \\ V9l2)))))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A.27a).(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V0l) = c_2Enum_2E0) \Leftrightarrow (\\
& V0l = (c_2Elist_2ENIL\ A.27a)))) \wedge ((\forall V1l \in (ty_2Elist_2Elist \\
& A.27a).(\forall V2n \in ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH \\
& A.27a)\ V1l) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& V2n))) \Leftrightarrow (\exists V3h \in A.27a.(\exists V4l_27 \in (ty_2Elist_2Elist \\
& A.27a).(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V4l_27) = (ap\ (ap\ c_2Earithmetic_2E_2D \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V2n))) \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \wedge \\
& (V1l = (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V3h)\ V4l_27)))))) \wedge ((\forall V5l \in \\
& (ty_2Elist_2Elist\ A.27a).(\forall V6n \in ty_2Enum_2Enum.(((ap \\
& (c_2Elist_2ELENGTH\ A.27a)\ V5l) = (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ V6n))) \Leftrightarrow (\exists V7h \in A.27a.(\exists V8l_27 \in \\
& (ty_2Elist_2Elist\ A.27a).(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V8l_27) = \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V6n))) \wedge \\
& (V5l = (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V7h)\ V8l_27)))))) \wedge (\forall V9l \in \\
& (ty_2Elist_2Elist\ A.27a).(\forall V10n1 \in ty_2Enum_2Enum.(\forall V11n2 \in \\
& ty_2Enum_2Enum.(((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V9l) = (ap\ (ap \\
& c_2Earithmetic_2E_2B\ V10n1)\ V11n2))) \Leftrightarrow (\exists V12l1 \in (ty_2Elist_2Elist \\
& A.27a).(\exists V13l2 \in (ty_2Elist_2Elist\ A.27a).(((ap\ (c_2Elist_2ELENGTH \\
& A.27a)\ V12l1) = V10n1) \wedge (((ap\ (c_2Elist_2ELENGTH\ A.27a)\ V13l2) = \\
& V11n2) \wedge (V9l = (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V12l1)\ V13l2))))))))))))) \\
& (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c_2Enum_2ESUC\ c_2Earithmetic_2EZERO) = (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum.((ap \\
& c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT2 \\
& V1n)) = (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enum_2ESUC\ V1n)))))) \\
& (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) (ap c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& (ap c_2Earithmic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1b \in 2. (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3m \in ty_2Enum_2Enum. (((ap (ap (ap c_2Enumeral_2EiSUB \\
& V1b) c_2Earithmetic_2EZERO) V0x) = c_2Earithmetic_2EZERO) \wedge (\\
& ((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) c_2Earithmetic_2EZERO) = \\
& V2n) \wedge (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) c_2Earithmetic_2EZERO) = (ap c_2Enumeral_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge ((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))))))))))))))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V0n) \\
& V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n)) (ap c_2Earithmetic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB \\
& c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0)))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap\ c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO)))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\forall V2l \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V2l))) \Leftrightarrow (\exists V3l1 \in (ty_2Elist_2Elist\ A_27a). (\exists V4l2 \in (ty_2Elist_2Elist\ A_27a). (((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V3l1) = V0n) \wedge ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V4l2)))) \wedge (V2l = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V3l1)\ V4l2)))))))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l))) \Leftrightarrow (\exists V2l1 \in (ty_2Elist_2Elist\ A_27a). (\exists V3l2 \in (ty_2Elist_2Elist\ A_27a). (((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V2l1) = V0n) \wedge (V1l = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V2l1)\ V3l2)))))))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_27a). ((c_2Enum_2E0 = (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l)) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_27a)))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_27a). (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_27a))) \wedge \\
& \quad (((p\ (ap\ (ap\ c_2Earithmetic_2E_3E\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l))) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_27a))) \wedge (((p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D\ c_2Enum_2E0)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l))) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_27a))) \wedge ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l))\ c_2Enum_2E0)) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_27a)))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).(\forall V1x \in ty_2Enum_2Enum.((((ap (c_2Elist_2ELENGTH \\
& A_27a) V0l) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \Leftrightarrow (\exists V2e1 \in \\
& A_27a.(\exists V3e2 \in A_27a.(\exists V4e3 \in A_27a.(\exists V5e4 \in \\
& A_27a.(V0l = (ap (ap (c_2Elist_2ECONS A_27a) V2e1) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V3e2) (ap (ap (c_2Elist_2ECONS A_27a) V4e3) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V5e4) (c_2Elist_2ENIL A_27a)))))))))) \wedge (((ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& (ap (c_2Elist_2ELENGTH A_27a) V0l)) \Leftrightarrow (\exists V6e1 \in A_27a.(\exists V7e2 \in \\
& A_27a.(\exists V8e3 \in A_27a.(\exists V9e4 \in A_27a.(V0l = (ap (ap \\
& (c_2Elist_2ECONS A_27a) V6e1) (ap (ap (c_2Elist_2ECONS A_27a) \\
& V7e2) (ap (ap (c_2Elist_2ECONS A_27a) V8e3) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V9e4) (c_2Elist_2ENIL A_27a)))))))))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) (ap (c_2Elist_2ELENGTH A_27a) V0l))) \Leftrightarrow \\
& (\exists V10l_27 \in (ty_2Elist_2Elist A_27a).(\exists V11e1 \in A_27a. \\
& (\exists V12e2 \in A_27a.(\exists V13e3 \in A_27a.(\exists V14e4 \in A_27a. \\
& (V0l = (ap (ap (c_2Elist_2ECONS A_27a) V11e1) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V12e2) (ap (ap (c_2Elist_2ECONS A_27a) V13e3) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V14e4) V10l_27)))))))))) \wedge (((p (ap (ap c_2Earithmetic_2E_3E \\
& (ap (c_2Elist_2ELENGTH A_27a) V0l)) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \Leftrightarrow \\
& (\exists V15l_27 \in (ty_2Elist_2Elist A_27a).(\exists V16e1 \in A_27a. \\
& (\exists V17e2 \in A_27a.(\exists V18e3 \in A_27a.(\exists V19e4 \in A_27a. \\
& (V0l = (ap (ap (c_2Elist_2ECONS A_27a) V16e1) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V17e2) (ap (ap (c_2Elist_2ECONS A_27a) V18e3) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V19e4) V15l_27)))))))))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) (ap (c_2Elist_2ELENGTH A_27a) V0l))) \Leftrightarrow \\
& (\exists V20l_27 \in (ty_2Elist_2Elist A_27a).(\exists V21e1 \in A_27a. \\
& (\exists V22e2 \in A_27a.(\exists V23e3 \in A_27a.(\exists V24e4 \in A_27a. \\
& (V0l = (ap (ap (c_2Elist_2ECONS A_27a) V21e1) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V22e2) (ap (ap (c_2Elist_2ECONS A_27a) V23e3) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V24e4) V20l_27)))))))))) \wedge (((p (ap (ap c_2Earithmetic_2E_3E_3D \\
& (ap (c_2Elist_2ELENGTH A_27a) V0l)) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \Leftrightarrow \\
& (\exists V25l_27 \in (ty_2Elist_2Elist A_27a).(\exists V26e1 \in A_27a. \\
& (\exists V27e2 \in A_27a.(\exists V28e3 \in A_27a.(\exists V29e4 \in A_27a. \\
& (V0l = (ap (ap (c_2Elist_2ECONS A_27a) V26e1) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V27e2) (ap (ap (c_2Elist_2ECONS A_27a) V28e3) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V29e4) V25l_27)))))))))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& V1x)) (ap (c_2Elist_2ELENGTH A_27a) V0l))) \Leftrightarrow (\exists V30l_27 \in \\
& (ty_2Elist_2Elist A_27a).(\exists V31e1 \in A_27a.(\exists V32e2 \in \\
& A_27a.(\exists V33e3 \in A_27a.(\exists V34e4 \in A_27a.((p (ap (ap \\
& c_2Earithmetic_2E_3C_3D V1x) (ap (c_2Elist_2ELENGTH A_27a) V30l_27))) \wedge \\
& (V0l = (ap (ap (c_2Elist_2ECONS A_27a) V31e1) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V32e2) (ap (ap (c_2Elist_2ECONS A_27a) V33e3) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V34e4) V30l_27)))))))))) \wedge (((p (ap (ap c_2Earithmetic_2E_3E_3D \\
& (ap (c_2Elist_2ELENGTH A_27a) V0l)) (ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) V1x))) \Leftrightarrow (\exists V35l_27 \in (ty_2Elist_2Elist \\
& A_27a).(\exists V36e1 \in A_27a.(\exists V37e2 \in A_27a.(\exists V38e3 \in \\
& A_27a.(\exists V39e4 \in A_27a.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1x) (ap (c_2Elist_2ELENGTH A_27a) V35l_27))) \wedge (V0l = (ap (ap (c_2Elist_2ECONS \\
& A_27a) V36e1) (ap (ap (c_2Elist_2ECONS A_27a) V37e2) (ap (ap (c_2Elist_2ECONS \\
& A_27a) V38e3) (ap (ap (c_2Elist_2ECONS A_27a) V39e4) V35l_27)))))))))) \wedge
\end{aligned}$$