thm_2EquantHeuristics_2ELIST__LENGTH__5 (TMRszzmUTHivkr8bAAqpaZ3eKoMT1eK947v)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{2}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{3}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{7}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{8}$$

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 11 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2E}$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Definition 12 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $c\_2Enumeral\_2EiSUB : \iota$ be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^2) \tag{12}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Definition 14 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 15 *We define* c_2Enumeral_2EiDUB *to be* $\lambda V0x \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{15}$$

Definition 16 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 17 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 18 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 19 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 20 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 21 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{16}$$

Definition 22 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Definition 23 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 24 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{17}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{18}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))))) \tag{19}$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0c)\ V0c) = \mathsf{c\_2Enum\_2E0})) \tag{20}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ V0n)\ V1m)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))))) \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge ((\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n)) \Leftrightarrow (\exists V3h \in A\_27a.(\exists V4l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l\_27) = V2n) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V4l\_27)))))))) \wedge (\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6n1 \in ty\_2Enum\_2Enum.(\forall V7n2 \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V5l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V6n1)\ V7n2)) \Leftrightarrow (\exists V8l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V9l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V8l1) = V6n1) \wedge (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V9l2) = V7n2) \wedge (V5l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V8l1)\ V9l2)))))))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge ((\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n))) \Leftrightarrow (\exists V3h \in A\_27a.(\exists V4l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l\_27) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V4l\_27)))))))) \wedge ((\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6n \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V5l) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V6n))) \Leftrightarrow (\exists V7h \in A\_27a.(\exists V8l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V8l\_27) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V6n))) \wedge (V5l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V7h)\ V8l\_27)))))))) \wedge (\forall V9l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V10n1 \in ty\_2Enum\_2Enum.(\forall V11n2 \in ty\_2Enum\_2Enum.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V9l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V10n1)\ V11n2)) \Leftrightarrow (\exists V12l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V13l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V12l1) = V10n1) \wedge (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V13l2) = V11n2) \wedge (V9l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V12l1)\ V13l2)))))))))))) \tag{30}$$

Assume the following.

$$(((ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Earithmetic\_2EZERO}) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})) \wedge ((\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \tag{31}$$

Assume the following.

$$((\forall V0n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}$$
$$(\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$$
$$\mathit{ty\_2Enum\_2Enum}.(\forall V3m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V3m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}$$
$$(\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V2n)\ V3m))))))\wedge((\forall V4n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge$$
$$((\forall V5n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}$$
$$V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in \mathit{ty\_2Enum\_2Enum}.$$
$$(\forall V7m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ ($$
$$\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V7m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}$$
$$V6n)\ V7m)))))\wedge((\forall V8n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}$$
$$\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$$
$$\mathit{ty\_2Enum\_2Enum}.(\forall V11m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V11m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}$$
$$V10n)\ V11m)))))\wedge((\forall V12n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}$$
$$\mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}$$
$$(\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$$
$$\mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ V14n)\ \mathsf{c\_2Enum\_2E0}) =$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge$$
$$((\forall V15n \in \mathit{ty\_2Enum\_2Enum}.(\forall V16m \in \mathit{ty\_2Enum\_2Enum}.$$
$$((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ V15n)\ V16m)))))\wedge(((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}$$
$$\mathsf{c\_2Enum\_2E0}) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in \mathit{ty\_2Enum\_2Enum}.($$
$$(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) =$$
$$\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}$$
$$(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in \mathit{ty\_2Enum\_2Enum}.$$
$$(((\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge$$
$$((\forall V20n \in \mathit{ty\_2Enum\_2Enum}.((\mathsf{c\_2Enum\_2E0} = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in \mathit{ty\_2Enum\_2Enum}.$$
$$(\forall V22m \in \mathit{ty\_2Enum\_2Enum}.(((\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V21n) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$$
$$((\forall V23n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow \mathit{False}))\wedge((\forall V24n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V24n)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V24n))))\wedge((\forall V25n \in \mathit{ty\_2Enum\_2Enum}.(\forall V26m \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V25n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V25n)\ V26m)))))\wedge((\forall V27n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}$$
$$\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow \mathit{False}))\wedge((\forall V28n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V28n))))\wedge((\forall V29n \in \mathit{ty\_2Enum\_2Enum}.(\forall V30m \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V29n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V30m)\ V29n)))))\wedge((\forall V31n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow \mathit{True}))\wedge((\forall V32n \in \mathit{ty\_2Enum\_2Enum}.$$
$$((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))) \tag{33}$$

Assume the following.

$$\begin{gathered}
(\forall V0x \in ty_2Enum_2Enum.(\forall V1b \in 2.(\forall V2n \in ty_2Enum_2Enum. \\
(\forall V3m \in ty_2Enum_2Enum.(((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB \\
V1b)\ \mathsf{c_2Earithmetic_2EZERO})\ V0x) = \mathsf{c_2Earithmetic_2EZERO}) \land (\\
((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ \mathsf{c_2Earithmetic_2EZERO}) = \\
V2n) \land (((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
V2n))\ \mathsf{c_2Earithmetic_2EZERO}) = (ap\ \mathsf{c_2Enumeral_2EiDUB}\ V2n)) \land \\
(((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V3m)) = (ap\ \mathsf{c_2Enumeral_2EiDUB} \\
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ V3m))) \land (((ap \\
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ V2n)\ V3m))) \land (((ap \\
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ V2n)\ V3m))) \land (((ap \\
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V3m)) = (ap\ \mathsf{c_2Enumeral_2EiDUB} \\
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ V2n)\ V3m))) \land (((ap \\
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2} \\
V2n))\ \mathsf{c_2Earithmetic_2EZERO}) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V2n)) \land \\
(((ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2} \\
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ V3m))) \land (((ap \\
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2} \\
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V3m)) = (ap\ \mathsf{c_2Enumeral_2EiDUB} \\
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ V3m))) \land (((ap \\
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2} \\
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V3m)) = (ap\ \mathsf{c_2Enumeral_2EiDUB} \\
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2ET})\ V2n)\ V3m))) \land ((ap \\
(ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2} \\
V2n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
(ap\ (ap\ (ap\ c_2Enumeral_2EiSUB\ \mathsf{c_2Ebool_2EF})\ V2n)\ V3m)))))))))))))))))
\end{gathered} \tag{34}$$

Assume the following.

$$\begin{gathered}
(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(\\
(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V0n) \\
V1m)) = (ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ ty_2Enum_2Enum)\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C} \\
V1m)\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ (ap\ (ap\ c_2Enumeral_2EiSUB \\
\mathsf{c_2Ebool_2ET})\ V0n)\ V1m)))\ \mathsf{c_2Enum_2E0}))))
\end{gathered} \tag{35}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ V0n)))\wedge(((ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))\wedge((ap\ \mathsf{c\_2Enumeral\_2EiDUB}\ \mathsf{c\_2Earithmetic\_2EZERO}) = \mathsf{c\_2Earithmetic\_2EZERO})))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\forall V2l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V2l))) \Leftrightarrow (\exists V3l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V4l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V3l1) = V0n)\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l2)))\wedge(V2l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V3l1)\ V4l2)))))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Leftrightarrow (\exists V2l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V3l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V2l1) = V0n)\wedge(V1l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2l1)\ V3l2)))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((\mathsf{c\_2Enum\_2E0} = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ \mathsf{c\_2Enum\_2E0})\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a)))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))\ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow (V0l = (c\_2Elist\_2ENIL\ A\_27a))))))) \tag{40}$$

Theorem 1

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
A\_27a).(\forall V1x \in ty\_2Enum\_2Enum.((((ap\ (c\_2Elist\_2ELENGTH \\
A\_27a)\ V0l) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \Leftrightarrow (\exists V2e1 \in \\
A\_27a.(\exists V3e2 \in A\_27a.(\exists V4e3 \in A\_27a.(\exists V5e4 \in \\
A\_27a.(\exists V6e5 \in A\_27a.(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a) \\
V2e1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3e2)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V4e3)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5e4)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V6e5)\ (c\_2Elist\_2ENIL\ A\_27a))))))))))))) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = \\
(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)) \Leftrightarrow (\exists V7e1 \in A\_27a.(\exists V8e2 \in \\
A\_27a.(\exists V9e3 \in A\_27a.(\exists V10e4 \in A\_27a.(\exists V11e5 \in \\
A\_27a.(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V7e1)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V8e2)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V9e3)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V10e4)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V11e5)\ (c\_2Elist\_2ENIL \\
A\_27a))))))))))))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \Leftrightarrow (\exists V12l\_27 \in (ty\_2Elist\_2Elist \\
A\_27a).(\exists V13e1 \in A\_27a.(\exists V14e2 \in A\_27a.(\exists V15e3 \in \\
A\_27a.(\exists V16e4 \in A\_27a.(\exists V17e5 \in A\_27a.(V0l = (ap\ (\\
ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V13e1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a) \\
V14e2)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V15e3)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V16e4)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V17e5)\ V12l\_27))))))))))))) \wedge \\
(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\
V0l))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \Leftrightarrow (\exists V18l\_27 \in \\
(ty\_2Elist\_2Elist\ A\_27a).(\exists V19e1 \in A\_27a.(\exists V20e2 \in \\
A\_27a.(\exists V21e3 \in A\_27a.(\exists V22e4 \in A\_27a.(\exists V23e5 \in \\
A\_27a.(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V19e1)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V20e2)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V21e3)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V22e4)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V23e5)\ V18l\_27))))))))))))) \wedge \\
(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \Leftrightarrow (\exists V24l\_27 \in (ty\_2Elist\_2Elist \\
A\_27a).(\exists V25e1 \in A\_27a.(\exists V26e2 \in A\_27a.(\exists V27e3 \in \\
A\_27a.(\exists V28e4 \in A\_27a.(\exists V29e5 \in A\_27a.(V0l = (ap\ (\\
ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V25e1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a) \\
V26e2)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V27e3)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V28e4)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V29e5)\ V24l\_27))))))))))))) \wedge \\
(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\
V0l))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \Leftrightarrow (\exists V30l\_27 \in \\
(ty\_2Elist\_2Elist\ A\_27a).(\exists V31e1 \in A\_27a.(\exists V32e2 \in \\
A\_27a.(\exists V33e3 \in A\_27a.(\exists V34e4 \in A\_27a.(\exists V35e5 \in \\
A\_27a.(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V31e1)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V32e2)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V33e3)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V34e4)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V35e5)\ V30l\_27))))))))))))) \wedge \\
(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
\mathsf{c\_2Earithmetic\_2EZERO}))))\ V1x))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a) \\
V0l))) \Leftrightarrow (\exists V36l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V37e1 \in \\
A\_27a.(\exists V38e2 \in A\_27a.(\exists V39e3 \in A\_27a.(\exists V40e4 \in \\
A\_27a.(\exists V41e5 \in A\_27a.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
V1x)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V36l\_27))) \wedge (V0l = (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V37e1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V38e2)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V39e3)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V40e4)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
A\_27a)\ V41e5)\ V36l\_27)))))))))))))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D} \\
(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}
\end{gathered}
$$