thm_2EquantHeuristics_2ELIST__LENGTH__COMPARE__SUC (TMby6N9jZnaph6ctnTAUyCSH5uysTLEpxA5)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 15 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{6}$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{7}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{8}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{9}$$

Definition 17 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 18 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 19 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{10}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\tag{11}$$

Assume the following.

$$True\tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))))\tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True))\tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))\tag{15}$$

Assume the following.

$$
\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist
A\_27a).(\forall V1x \in ty\_2Enum\_2Enum.((((ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V0l) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}
\mathsf{c\_2Earithmetic\_2EZERO}))) \Leftrightarrow (\exists V2e1 \in A\_27a.(V0l = (ap\ (ap
(c\_2Elist\_2ECONS\ A\_27a)\ V2e1)\ (c\_2Elist\_2ENIL\ A\_27a))))) \wedge ((
((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})) =
(ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)) \Leftrightarrow (\exists V3e1 \in A\_27a.(V0l =
(ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3e1)\ (c\_2Elist\_2ENIL\ A\_27a))))) \wedge
(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V0l))) \Leftrightarrow (\exists V4l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V5e1 \in
A\_27a.(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5e1)\ V4l\_27))))) \wedge
(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)
V0l))\ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow (\exists V6l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).
(\exists V7e1 \in A\_27a.(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V7e1)
V6l\_27))))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V0l))) \Leftrightarrow (\exists V8l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V9e1 \in
A\_27a.(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V9e1)\ V8l\_27))))) \wedge
(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)
V0l))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}
\mathsf{c\_2Earithmetic\_2EZERO})))) \Leftrightarrow (\exists V10l\_27 \in (ty\_2Elist\_2Elist
A\_27a).(\exists V11e1 \in A\_27a.(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)
V11e1)\ V10l\_27))))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap
(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}
\mathsf{c\_2Earithmetic\_2EZERO})))\ V1x))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)
V0l))) \Leftrightarrow (\exists V12l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V13e1 \in
A\_27a.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1x)\ (ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V12l\_27))) \wedge (V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V13e1)
V12l\_27)))))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ (ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V0l))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V1x))) \Leftrightarrow
(\exists V14l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V15e1 \in A\_27a.
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1x)\ (ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V14l\_27))) \wedge (V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V15e1)
V14l\_27)))))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B
V1x)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}
\mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \Leftrightarrow
(\exists V16l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V17e1 \in A\_27a.
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1x)\ (ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V16l\_27))) \wedge (V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V17e1)
V16l\_27)))))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ (ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V0l))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1x)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \Leftrightarrow (\exists V18l\_27 \in
(ty\_2Elist\_2Elist\ A\_27a).(\exists V19e1 \in A\_27a.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}
V1x)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V18l\_27))) \wedge (V0l = (ap\ (ap\ (c\_2Elist\_2ECONS
A\_27a)\ V19e1)\ V18l\_27)))))) \wedge ((((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)
V0l) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V1x)) \Leftrightarrow (
\exists V20l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V21e1 \in A\_27a.
(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V20l\_27) = V1x) \wedge (V0l = (ap\ (ap\ (
c\_2Elist\_2ECONS\ A\_27a)\ V21e1)\ V20l\_27)))))) \wedge ((((ap\ (ap\ c\_2Earithmetic\_2E\_2B
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))
V1x) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)) \Leftrightarrow (\exists V22l\_27 \in (
ty\_2Elist\_2Elist\ A\_27a).(\exists V23e1 \in A\_27a.(((ap\ (c\_2Elist\_2ELENGTH
A\_27a)\ V22l\_27) = V1x) \wedge (V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V23e1)
V22l\_27)))))) \wedge ((((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l) = (ap\ (ap
c\_2Earithmetic\_2E\_2B\ V1x)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}
\mathsf{c\_2Earithmetic\_2EZERO})))) \Leftrightarrow (\exists V24l\_27 \in (ty\_2Elist\_2Elist
A\_27a).(\exists V25e1 \in A\_27a.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)
$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in ty\_2Enum\_2Enum.(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0x))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Leftrightarrow (\exists V2l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V3e1 \in A\_27a.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0x)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V2l\_27))) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3e1)\ V2l\_27)))))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0x))) \Leftrightarrow (\exists V4l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V5e1 \in A\_27a.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0x)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l\_27))) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5e1)\ V4l\_27)))))) \wedge ((((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0x)) \Leftrightarrow (\exists V6l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V7e1 \in A\_27a.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V6l\_27) = V0x) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V7e1)\ V6l\_27)))))) \wedge (((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0x) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l)) \Leftrightarrow (\exists V8l\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V9e1 \in A\_27a.(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V8l\_27) = V0x) \wedge (V1l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V9e1)\ V8l\_27)))))))))))$$