

# thm\_2EquantHeuristics\_2ESIMPLE\_GUESS\_EXISTS\_ALT\_DEF (TMQXAci5vE2nzg5M25u8zr773aJsLfUei2r)

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Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E2$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21 2)) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 7** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E40 A\_27a P))))$

**Definition 9** We define  $c\_2EquantHeuristics\_2EGUESS\_ EXISTS\_ GAP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (2^{A\_27a})$

**Definition 10** We define  $c\_2EquantHeuristics\_2ESIMPLE\_ GUESS\_ EXISTS$  to be  $\lambda A\_27a : \iota.\lambda V0v \in A\_27a.\lambda V1i \in A\_27a.\lambda V2P \in 2.(ap (ap (c\_2Emin\_2E3D\_3D\_3E V2P) (ap (ap (c\_2Emin\_2E3D (2^{A\_27a}))$

Assume the following.

$$True \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (4)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (6)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap\ (ap\ (c\_2Ecombin\_2EK\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \quad (8)$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0i \in A\_27a. (\forall V1P \in (2^{A\_27a}). ((\forall V2v \in A\_27a. (p\ (ap\ (ap\ (ap\ (c\_2EquantHeuristics\_2ESIMPLE\_GUESS\_EXISTS\ A\_27a)\ V2v)\ V0i)\ (ap\ V1P\ V2v)))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2EquantHeuristics\_2EGUESS\_EXISTS\_GAP\ ty\_2Eone\_2Eone\ A\_27a)\ (ap\ (c\_2Ecombin\_2EK\ A\_27a\ ty\_2Eone\_2Eone)\ V0i))\ (\lambda V3v \in A\_27a. (ap\ V1P\ V3v)))))))$$