

thm_2EquantHeuristics_2ESND__PAIR__EQ (TMTRhzwJbkE1xJgGaG8zwwjG7GgxaqsG4iTL)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A) (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2Efst : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2Efst A_27a A_27b \in (A_27a (ty_2Epair_2Eprod A_27a A_27b)) \tag{2}$$

Let $c_2Epair_2Eabs_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2Eabs_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \tag{3}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2ESND \\ A.27a A.27b \in (A.27b^{(ty_2Epair_2Eprod A.27a A.27b)}) \end{aligned} \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A.27a.(p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\ A.27b.(((ap (ap (c_2Epair_2E_2C A.27a A.27b) V0x) V1y) = (ap (ap \\ (c_2Epair_2E_2C A.27a A.27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ \forall V0x \in (ty_2Epair_2Eprod A.27a A.27b).(\exists V1q \in A.27a. \\ (\exists V2r \in A.27b.(V0x = (ap (ap (c_2Epair_2E_2C A.27a A.27b) \\ V1q) V2r)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap (c_2Epair_2EFST A.27a \\ A.27b) (ap (ap (c_2Epair_2E_2C A.27a A.27b) V0x) V1y)) = V0x))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap (c_2Epair_2ESND A.27a \\ A.27b) (ap (ap (c_2Epair_2E_2C A.27a A.27b) V0x) V1y)) = V1y))) \end{aligned} \quad (12)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0p \in (ty_2Epair_2Eprod\ A.27a\ A.27b). (\forall V1p1 \in A.27a. \\ & \quad ((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V1p1)\ (ap\ (c_2Epair_2ESND \\ & \quad A.27a\ A.27b)\ V0p)) = V0p) \Leftrightarrow (V1p1 = (ap\ (c_2Epair_2EFST\ A.27a\ A.27b) \\ & \quad V0p)))))) \end{aligned}$$