

thm_2EquantHeuristics_2ESND__PAIR__EQ__SYM (TMHo14obzrSfD1tBbtwkrVXks1Y5t8j4at4)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 0t \in 2. V 0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 2t \in 2. V 2t))))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } V 0P (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 A 1) \tag{1}$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EFST } A. 27a A. 27b \in (A. 27a (\text{ty_2Epair_2Eprod } A. 27a A. 27b)) \tag{2}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EABS_prod } A. 27a A. 27b \in ((\text{ty_2Epair_2Eprod } A. 27a A. 27b)^{(2^{A-27b})^{A-27a}}) \tag{3}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND \\ A.27a\ A.27b \in (A.27b^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \end{aligned} \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A.27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\ A.27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b).(\exists V1q \in A.27a. \\ (\exists V2r \in A.27b.(V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b) \\ V1q)\ V2r)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c_2Epair_2EFST\ A.27a \\ A.27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c_2Epair_2ESND\ A.27a \\ A.27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (12)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow (\\ & \quad \forall V0p \in (\text{ty_2Epair_2Eprod } A_{27a} \ A_{27b}). (\forall V1p1 \in A_{27a}. \\ & \quad ((V0p = (\text{ap } (\text{ap } (\text{c_2Epair_2E_2C } A_{27a} \ A_{27b}) \ V1p1) \ (\text{ap } (\text{c_2Epair_2ESND} \\ & \quad A_{27a} \ A_{27b}) \ V0p))) \Leftrightarrow ((\text{ap } (\text{c_2Epair_2EFST } A_{27a} \ A_{27b}) \ V0p) = V1p1)))) \end{aligned}$$