

# thm\_2EquantHeuristics\_2EUNWIND\_EXISTS\_THM (TMFjj8qkTUTUsAHcJ3nJHBqx Bq5zSjhVLha)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p (ap P x))$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A P))))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) P) V0P))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 9** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (4)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ & A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in \\ & A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p (ap V0P V2x)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in \\ & A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p (ap V0P V2x)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ & (2^{A\_27a}).((\exists V2x \in A\_27a.((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow \\ & ((\exists V3x \in A\_27a.(p (ap V0P V3x))) \vee (\exists V4x \in A\_27a.(p ( \\ & ap V1Q V4x)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ & 2.(((\exists V2x \in A\_27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in \\ & A\_27a.((p (ap V0P V3x)) \vee (p V1Q)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ & 2.((\exists V2x \in A\_27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A\_27a.(p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee (p \ V0A)))))) \quad (15)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \vee \neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \wedge \neg(p \ V1B)))))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow \text{False}))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \ V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p \ V1B)) \Rightarrow \text{False})))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow \text{False})))))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow \text{False}) \Rightarrow (((p \ V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (21)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(p \ V2r)) \vee \neg(p \ V1q))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee \neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee \neg(p \ V0p)))))))))) \quad (22)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee \neg(p \ V2r))) \wedge (((p \ V1q) \vee \neg(p \ V0p)) \wedge ((p \ V2r) \vee \neg(p \ V0p)))))))) \quad (23)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee \neg(p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge (( \\
& \neg(p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow \neg(p \vee 1q)) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge (\neg(p \vee 1q) \vee \neg(p \vee 0p))))))
\end{aligned} \tag{26}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p))) \tag{27}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow \neg(p \vee 1q))) \tag{28}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow \neg(p \vee 0p))) \tag{29}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow \neg(p \vee 1q))) \tag{30}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p \vee 0p))) \Rightarrow (p \vee 0p)) \tag{31}$$

**Theorem 1**

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0a \in A_{27a}. (\forall V1P \in \\
& (2^{A_{27a}}). ((\exists V2x \in A_{27a}. (p \text{ (ap } V1P \text{ } V2x))) \Leftrightarrow ((\forall V3x \in \\
& A_{27a}. (\neg(V3x = V0a)) \Rightarrow \neg(p \text{ (ap } V1P \text{ } V3x)))))) \Rightarrow (p \text{ (ap } V1P \text{ } V0a))))))
\end{aligned}$$