thm_2Equote_2Ecompare__list__index (TMboNjPm6f2V9prqJpjkUHzBLjTjNGUuN4a)

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Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \quad (1)$$

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \quad (2)$$

Let $c\_2Equote\_2Eindex\_\_compare : \iota$ be given. Assume the following.

$$c\_2Equote\_2Eindex\_\_compare \in ((ty\_2EternaryComparisons\_2Eordering^{ty\_2Equote\_2Eindex})^{ty\_2Equote\_2E} \quad (3)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (4)$$

Let $c\_2EternaryComparisons\_2Elist\_\_compare : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EternaryComparisons$$
$$A\_27a\ A\_27b \in (((ty\_2EternaryComparisons\_2Eordering^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((ty} \quad (5)$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \quad (6)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Assume the following.

$$(\forall V0i1 \in ty\_2Equote\_2Eindex.(\forall V1i2 \in ty\_2Equote\_2Eindex. (((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare\ V0i1)\ V1i2) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow (V0i1 = V1i2)))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0cmp \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}). ((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((ap\ (ap\ V0cmp\ V1x)\ V2y) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow (V1x = V2y)))) \Rightarrow (\forall V3l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V4l2 \in (ty\_2Elist\_2Elist\ A\_27a). (((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare\ A\_27a\ A\_27a)\ V0cmp)\ V3l1)\ V4l2) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow (V3l1 = V4l2)))))) \tag{8}$$

Theorem 1

$$(\forall V0l1 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V1l2 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_compare\ ty\_2Equote\_2Eindex\ ty\_2Equote\_2Eindex)\ c\_2Equote\_2Eindex\_\_compare)\ V0l1)\ V1l2) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow (V0l1 = V1l2))))$$