

thm_2Equotient_2EC__PRS (TMFW_{vvQE9EanFNM4taaaf3NZ3t4oqqi5Yoh})

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ecombin_2EC` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a})) . (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a})))$

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 7 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27b}) . \lambda V0f$

Definition 8 We define `c_2Equotient_2E_2D_2D_3E` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0f$

Assume the following.

$$\text{True} \tag{1}$$

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$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \text{True})) \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \forall A_27c. \\ & \text{nonempty } A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\ & A_27b. (\forall V2y \in A_27a. ((\text{ap } (\text{ap } (\text{ap } (\text{c_2Ecombin_2EC } A_27a } A_27b \\ & A_27c) V0f) V1x) V2y) = (\text{ap } (\text{ap } V0f V2y) V1x)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3a \in A_27b. ((ap\ V1abs \\
& \quad (ap\ V2rep\ V3a)) = V3a))))))
\end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27c^{A_27a}). \\
& \quad (\forall V1g \in (A_27d^{A_27b}). (\forall V2h \in (A_27b^{A_27c}). (\forall V3x \in \\
& A_27a. ((ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27a\ A_27b\ A_27c \\
& A_27d)\ V0f)\ V1g)\ V2h)\ V3x) = (ap\ V1g\ (ap\ V2h\ (ap\ V0f\ V3x)))))))))
\end{aligned} \tag{5}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow \forall A_27e.nonempty \\
& A_27e \Rightarrow \forall A_27f.nonempty\ A_27f \Rightarrow (\forall V0R1 \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1abs1 \in (A_27d^{A_27a}). (\forall V2rep1 \in (A_27a^{A_27d}). \\
& ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27d)\ V0R1)\ V1abs1) \\
& \quad V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in (\\
& A_27e^{A_27b}). (\forall V5rep2 \in (A_27b^{A_27e}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27e)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6R3 \in ((2^{A_27c})^{A_27c}). \\
& \quad (\forall V7abs3 \in (A_27f^{A_27c}). (\forall V8rep3 \in (A_27c^{A_27f}). \\
& ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27c\ A_27f)\ V6R3)\ V7abs3) \\
& \quad V8rep3))) \Rightarrow (\forall V9f \in ((A_27f^{A_27e})^{A_27d}). (\forall V10x \in A_27e. \\
& \quad (\forall V11y \in A_27d. ((ap\ (ap\ (ap\ (c_2Ecombin_2EC\ A_27d\ A_27e\ A_27f) \\
& V9f)\ V10x)\ V11y) = (ap\ V7abs3\ (ap\ (ap\ (ap\ (c_2Ecombin_2EC\ A_27a\ A_27b \\
& \quad A_27c)\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27a\ (A_27f^{A_27e}) \\
& \quad A_27d\ (A_27c^{A_27b}))\ V1abs1)\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27b\ A_27f\ A_27e\ A_27c)\ V4abs2)\ V8rep3))\ V9f))\ (ap\ V5rep2\ V10x)) \\
& \quad (ap\ V2rep1\ V11y)))))))))))))
\end{aligned}$$