thm_2Equotient_2EC__RSP (TM-LEVrz9vPL4UEi2niVdVSsbFRTQYpSZnbP)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V$

Definition 7 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27$

Definition 8 *We define* c_2Equotient_2E_3D_3D_3D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27a}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27b.(\forall V2y \in A\_27a.((ap\ (ap\ (ap\ (\text{c\_2Ecombin\_2EC}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1x)\ V2y) = (ap\ (ap\ V0f\ V2y)\ V1x))))) \tag{1}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty$$
$$A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).$$
$$(\forall V1abs1 \in (A\_27d^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27d}).$$
$$((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27d)\ V0R1)\ V1abs1)$$
$$V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in ($$
$$A\_27e^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27e}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}$$
$$A\_27b\ A\_27e)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6R3 \in ((2^{A\_27c})^{A\_27c}).$$
$$(\forall V7abs3 \in (A\_27f^{A\_27c}).(\forall V8rep3 \in (A\_27c^{A\_27f}).$$
$$((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27c\ A\_27f)\ V6R3)\ V7abs3)$$
$$V8rep3)) \Rightarrow (\forall V9f1 \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V10f2 \in$$
$$((A\_27c^{A\_27b})^{A\_27a}).(\forall V11x1 \in A\_27b.(\forall V12x2 \in A\_27b.$$
$$(\forall V13y1 \in A\_27a.(\forall V14y2 \in A\_27a.(((p\ (ap\ (ap\ (ap\ (ap$$
$$(\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ (A\_27c^{A\_27b}))\ V0R1)\ (ap\ ($$
$$ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27b\ A\_27c)\ V3R2)\ V6R3))\ V9f1)$$
$$V10f2)) \wedge ((p\ (ap\ (ap\ V3R2\ V11x1)\ V12x2)) \wedge (p\ (ap\ (ap\ V0R1\ V13y1)\ V14y2)))) \Rightarrow$$
$$(p\ (ap\ (ap\ V6R3\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27b\ A\_27c)\ V9f1)$$
$$V11x1)\ V13y1))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27b\ A\_27c)\ V10f2)$$
$$V12x2)\ V14y2))))))))))))))))))))$$