

thm_2Equotient_2EEQUIV_RES_EXISTS (TMM6PSeBeJwSoxPkJw9R1p5yyTPRzV2uCxB)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a})) (\lambda V1P \in (2^{A_27a}). V1P)) (\lambda V2P \in (2^{A_27a}). V2P)))$

Definition 5 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 6 We define `c_2Equotient_2EEQUIV` to be $\lambda A_27a : \iota. \lambda V0E \in ((2^{A_27a})^{A_27a}). (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V1E \in ((2^{A_27a})^{A_27a}). V1E))$

Definition 7 We define `c_2Ecombin_2EW` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in ((A_27b)^{A_27a})^{A_27a}). (\lambda V1x \in (A_27b)^{A_27a}. V1x)$

Definition 8 We define `c_2Equotient_2ERespects` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\text{c_2Ecombin_2EW } A_27a A_27b)$

Definition 9 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } V1f V0x)))$

Definition 10 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 11 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A_27a) (\lambda V1P \in (2^{A_27a}). V1P))))$

Definition 13 We define `c_2Ebool_2ERES_EXISTS` to be $\lambda A_27a : \iota. (\lambda V0p \in (2^{A_27a}). (\lambda V1m \in (2^{A_27a}). (\text{ap } V1m (\text{ap } (\text{c_2Emin_2E_40 } A_27a) (\lambda V2m \in (2^{A_27a}). V2m))))))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((\text{True} \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge \text{True}) \Leftrightarrow \\ & (p \ V0t)) \wedge (((\text{False} \wedge (p \ V0t)) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (3)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ V0P)) \Leftrightarrow (p\ (ap\ V0P\ V1x)))))) \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1x \in A_27a. ((p\ (ap\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R)\ V1x)) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V1x)\ V1x)))))) \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1f \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2ERES_EXISTS\ A_27a)\ V0P)\ V1f)) \Leftrightarrow (\exists V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0P)) \wedge (p\ (ap\ V1f\ V2x)))))))))) \quad (7)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). (\forall V1P \in (2^{A_27a}). ((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a)\ V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_EXISTS\ A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_3F\ A_27a)\ V1P)))))))$$