

thm_2Equotient_2EEQUIV__RES__EXISTS__UNIQUE
 (TMQPN-
 hqyc6tELHTibdFUcE6Tk9RvcCgh5mM)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F_5C$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F_5C$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))$

Definition 10 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 11 We define $c_2Equotient_2EEQUIV$ to be $\lambda A_27a : \iota.\lambda V0E \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_3F$

Definition 12 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x \in A_27a.V1x$

Definition 13 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b)$

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 15 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap V1m p))$

Definition 16 We define $c_2Ebool_2ERES_EXISTS$ to be $\lambda A_27a : \iota. (\lambda V0p \in (2^{A_27a}). (\lambda V1m \in (2^{A_27a}). ($

Definition 17 We define $c_2Ebool_2ERES_EXISTS_UNIQUE$ to be $\lambda A_27a : \iota. (\lambda V0p \in (2^{A_27a}). (\lambda V1m \in ($

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{4}$$

Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((p \ (ap \\ & (c_2Ebool_2E_3F_21 \ A_27a) \ (\lambda V1x \in A_27a. (ap \ V0P \ V1x))) \Leftrightarrow ((\\ & \exists V2x \in A_27a. (p \ (ap \ V0P \ V2x))) \wedge (\forall V3x \in A_27a. (\forall V4y \in \\ & A_27a. (((p \ (ap \ V0P \ V3x)) \wedge (p \ (ap \ V0P \ V4y))) \Rightarrow (V3x = V4y)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1x \in A_27a. ((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V1x) \ V0P)) \Leftrightarrow (p \ (ap \ V0P \ V1x)))))) \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ & (\forall V1x \in A_27a. ((p \ (ap \ (ap \ (c_2Equotient_2ERespects \ A_27a \ 2) \ V0R) \ V1x)) \Leftrightarrow (p \ (ap \ (ap \ V0R \ V1x) \ V1x)))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1f \in \\ & (2^{A_27a}). ((p \ (ap \ (ap \ (c_2Ebool_2ERES_FORALL \ A_27a) \ V0P) \ V1f)) \Leftrightarrow \\ & (\forall V2x \in A_27a. ((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V2x) \ V0P)) \Rightarrow \\ & (p \ (ap \ V1f \ V2x)))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1f \in \\ (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2ERES_EXISTS\ A.27a)\ V0P)\ V1f))) \Leftrightarrow \\ (\exists V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0P)) \wedge \\ (p\ (ap\ V1f\ V2x)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1f \in \\ (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2ERES_EXISTS_UNIQUE\ A.27a)\ V0P)\ V1f))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool.2ERES_EXISTS\ A.27a)\ V0P)\ (\lambda V2x \in \\ A.27a.(ap\ V1f\ V2x)))) \wedge (p\ (ap\ (ap\ (c.2Ebool.2ERES_FORALL\ A.27a)\ V0P)\ (\lambda V3x \in A.27a.(ap\ (ap\ (c.2Ebool.2ERES_FORALL\ A.27a)\ V0P)\ \\ (\lambda V4y \in A.27a.(ap\ (ap\ c.2Emin.2E.3D.3D.3E\ (ap\ (ap\ c.2Ebool.2E.2F.5C \\ (ap\ V1f\ V3x)\ (ap\ V1f\ V4y)))) (ap\ (ap\ (c.2Emin.2E.3D\ A.27a)\ V3x)\ V4y)))))))))) \end{aligned} \quad (11)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0E \in ((2^{A.27a})^{A.27a}). \\ (\forall V1P \in (2^{A.27a}). ((p\ (ap\ (c.2Equotient.2EEQUIV\ A.27a)\ V0E))) \Rightarrow ((p\ (ap\ (ap\ (c.2Ebool.2ERES_EXISTS_UNIQUE\ A.27a)\ (ap \\ (c.2Equotient.2Erespects\ A.27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c.2Ebool.2E.3F.21 \\ A.27a)\ V1P)))))) \end{aligned}$$