

thm_2Equotient_2EEXISTS__PRS
(TMawDyFqWN-
WCv9jtp9dBe1BULpTwS43CdSG)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 5 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t)).$

Definition 6 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type $\iota.$

Definition 7 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2EF})))$

Definition 8 We define `c_2Ecombin_2EK` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V0x \in A. 27a. (\lambda V1y \in A. 27b. V0x))$

Definition 9 We define `c_2Ecombin_2ES` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A. 27c^{A-27b})^{A-27a}))$

Definition 10 We define `c_2Ecombin_2EI` to be $\lambda A. 27a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A. 27a (A. 27a^{A-27a}))))$

Definition 11 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 12 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}).$

Definition 13 We define `c_2Equotient_2E_2D_2D_3E` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. \lambda A. 27d : \iota. \lambda V0f \in ((A. 27d^{A-27c})^{A-27b}).$

Definition 14 We define `c_2Ecombin_2EW` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V0f \in ((A. 27b^{A-27a})^{A-27a}). (\lambda V1x \in A. 27a. (\lambda V2y \in A. 27b. V0f)))$

Definition 15 We define `c_2Equotient_2Erespects` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\text{c_2Ecombin_2EW } A. 27a A. 27b)$

Definition 16 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define $c_2Ebool_2ERES_EXISTIS$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).($

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (1)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (2)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (3)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in ((A_27b^{A_27a})^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Ecombin_2EW\ A_27a\ A_27b)\ V0f)\ V1x) = (ap\ (ap\ V0f\ V1x)\ V1x)))) \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ V0P)) \Leftrightarrow (p\ (ap\ V0P\ V1x)))))) \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}).(\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3a \in A_27b.((ap\ V1abs\ (ap\ V2rep\ V3a)) = V3a)))))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}).(\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3a \in A_27b.(p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3a))\ (ap\ V2rep\ V3a)))))) \quad (8)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27c^{A_27a}). \\
& (\forall V1g \in (A_27d^{A_27b}). (\forall V2h \in (A_27b^{A_27c}). (\forall V3x \in \\
& A_27a. ((ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27a\ A_27b\ A_27c \\
& A_27d)\ V0f)\ V1g)\ V2h)\ V3x) = (ap\ V1g\ (ap\ V2h\ (ap\ V0f\ V3x)))))))))
\end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1f \in \\
& (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2ERES_EXISTS\ A_27a)\ V0P)\ V1f))) \Leftrightarrow \\
& (\exists V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0P)) \wedge \\
& (p\ (ap\ V1f\ V2x))))))
\end{aligned} \tag{10}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}). ((p\ (\\
& ap\ (c_2Ebool_2E_3F\ A_27b)\ V3f))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_EXISTS \\
& A_27a)\ (ap\ (c_2Equotient_2ERespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& 2))\ V3f)))))))))
\end{aligned}$$