

thm_2Equotient_2EFUN__QUOTIENT (TMJw9B32sUdJJTaH7CTc9GhkF6cPgDxhuZR)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ecombin_2EK` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. (\lambda V0x \in A. \lambda V1y \in A. \lambda V2z \in A. \text{c_2Emin_2E_3D } (V0x = V1y) \wedge V2z = V0x)$

Definition 4 We define `c_2Ecombin_2ES` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda A. \lambda c : \iota. (\lambda V0f \in ((A \rightarrow A)^{A \rightarrow A})) (\lambda V1x \in A. \lambda V2y \in A. \text{c_2Ecombin_2EK } (V0f \text{ c_2Emin_2E_3D } (V1x = V2y)))$

Definition 5 We define `c_2Ecombin_2EI` to be $\lambda A. \lambda a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A. \lambda a. (A \rightarrow A)^{A \rightarrow A})) A. \lambda a. (A \rightarrow A)^{A \rightarrow A})$

Definition 6 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 7 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A \rightarrow A})) (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A \rightarrow A})) (\lambda V1t \in 2. V1t)) (\lambda V2t \in 2. V2t))$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t)) (\lambda V3t \in 2. V3t)))$

Definition 9 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda V0R \in ((2^{A \rightarrow A})^{A \rightarrow A}). \lambda V1x \in A. \lambda V2y \in A. \text{c_2Emin_2E_3D } (V1x = V2y)$

Definition 10 We define `c_2Equotient_2E_2D_2D_3E` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda A. \lambda c : \iota. \lambda A. \lambda d : \iota. \lambda V0f \in ((A \rightarrow A)^{A \rightarrow A}) (\lambda V1x \in A. \lambda V2y \in A. \lambda V3z \in A. \text{c_2Equotient_2EQUOTIENT } (V0f \text{ c_2Emin_2E_3D } (V1x = V2y) \wedge V3z = V1x))$

Definition 11 We define `c_2Equotient_2E_3D_3D_3D_3E` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda V0R1 \in ((2^{A \rightarrow A})^{A \rightarrow A}) (\lambda V1x \in A. \lambda V2y \in A. \lambda V3z \in A. \lambda V4w \in A. \text{c_2Equotient_2EQUOTIENT } (V0R1 \text{ c_2Emin_2E_3D } (V1x = V2y) \wedge V3z = V4w))$

Definition 12 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 13 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t)) (\lambda V3t \in 2. V3t)))$

Definition 14 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\lambda V1t \in 2. V1t)) (\lambda V2t \in 2. V2t)))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \Rightarrow V0t1) \Rightarrow (p \Rightarrow V1t2)) \Rightarrow (((p \Rightarrow V1t2) \Rightarrow (p \Rightarrow V0t1)) \Rightarrow ((p \Rightarrow V0t1) \Leftrightarrow (p \Rightarrow V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x))))))) \quad (15)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (16)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. ((ap (c.2Ecombin_2El A.27a) V0x) = V0x)) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). (\forall V1abs \in (A.27b^{A.27a}). (\forall V2rep \in (A.27a^{A.27b}). ((p (ap (ap (ap (c.2Equotient_2EQUOTIENT A.27a A.27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3a \in A.27b. ((ap V1abs (ap V2rep V3a)) = V3a)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). (\forall V1abs \in (A.27b^{A.27a}). (\forall V2rep \in (A.27a^{A.27b}). ((p (ap (ap (ap (c.2Equotient_2EQUOTIENT A.27a A.27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3a \in A.27b. (p (ap (ap V0R (ap V2rep V3a)) (ap V2rep V3a)))))))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3r \in A_27a. (\forall V4s \in \\
& A_27a. ((p\ (ap\ (ap\ V0R\ V3r)\ V4s)) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V3r)\ V3r)) \wedge ((p\ (ap\ (ap\ V0R\ V4s)\ V4s)) \wedge ((ap\ V1abs\ V3r) = (ap\ V1abs\ V4s)))))))))) \\
& \hspace{15em} (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3r \in A_27a. (\forall V4s \in \\
& A_27a. ((p\ (ap\ (ap\ V0R\ V3r)\ V4s)) \Rightarrow ((ap\ V1abs\ V3r) = (ap\ V1abs\ V4s)))))))))) \\
& \hspace{15em} (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3r \in A_27a. ((p\ (ap\ (ap\ V0R\ V3r)\ V3r)) \Rightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ (ap\ V1abs\ V3r)))\ V3r)))))) \\
& \hspace{15em} (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27c^{A_27a}). \\
& \quad (\forall V1g \in (A_27d^{A_27b}). (\forall V2h \in (A_27b^{A_27c}). (\forall V3x \in \\
& A_27a. ((ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27a\ A_27b\ A_27c \\
& \quad A_27d)\ V0f)\ V1g)\ V2h)\ V3x) = (ap\ V1g\ (ap\ V2h\ (ap\ V0f\ V3x))))))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{10em} (26)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \hspace{10em} (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \hspace{10em} (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \hspace{10em} (29)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge (\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (35)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow \forall A_27d.nonempty A_27d \Rightarrow (\forall V0R1 \in (\\ & (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\ & (A_27a^{A_27c}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_27a A_27c) \\ & V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\ & (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\ & A_27b A_27d) V3R2) V4abs2) V5rep2)) \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\ & (A_27b^{A_27a}) (A_27d^{A_27c})) (ap (ap (c_2Equotient_2E_3D_3D_3D_3E \\ & A_27a A_27b) V0R1) V3R2)) (ap (ap (c_2Equotient_2E_2D_2D_3E A_27c \\ & A_27b A_27a A_27d) V2rep1) V4abs2)) (ap (ap (c_2Equotient_2E_2D_2D_3E \\ & A_27a A_27d A_27c A_27b) V1abs1) V5rep2)))))))))) \end{aligned}$$