

thm_2Equotient_2EFUN_REL_EQ_REL
(TMde3kw4AtVDtJF5vunRoZABt137g2dvrxB)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)$

Definition 8 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f$

Definition 9 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27a})$

Definition 10 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda$

Definition 11 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x$

Definition 12 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b)$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& (p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0f \in ((A_27b^{A_27a})^{A_27a}).(\forall V1x \in A_27a.((ap \ (ap \\
& (c_2Ecombin_2EW \ A_27a \ A_27b) \ V0f) \ V1x) = (ap \ (ap \ V0f \ V1x) \ V1x))))
\end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p \ (ap \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \\
& A_27a \ A_27b) \ V0R) \ V1abs) \ V2rep)) \Rightarrow (\forall V3r \in A_27a.(\forall V4s \in \\
& A_27a.((p \ (ap \ (ap \ V0R \ V3r) \ V4s)) \Leftrightarrow ((p \ (ap \ (ap \ V0R \ V3r) \ V3r)) \wedge ((p \ (ap \\
& (ap \ V0R \ V4s) \ V4s)) \wedge ((ap \ V1abs \ V3r) = (ap \ V1abs \ V4s))))))))))
\end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow \forall A_27c. \\
& nonempty \ A_27c \Rightarrow \forall A_27d.nonempty \ A_27d \Rightarrow (\forall V0R1 \in (\\
& (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& (A_27a^{A_27c}).((p \ (ap \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \ A_27a \ A_27c) \\
& V0R1) \ V1abs1) \ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p \ (ap \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \\
& A_27b \ A_27d) \ V3R2) \ V4abs2) \ V5rep2)) \Rightarrow (p \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \\
& (A_27b^{A_27a}) \ (A_27d^{A_27c})) \ (ap \ (ap \ (c_2Equotient_2E_3D_3D_3D_3E \\
& A_27a \ A_27b) \ V0R1) \ V3R2)) \ (ap \ (ap \ (c_2Equotient_2E_2D_2D_3E \ A_27c \\
& A_27b \ A_27a \ A_27d) \ V2rep1) \ V4abs2)) \ (ap \ (ap \ (c_2Equotient_2E_2D_2D_3E \\
& A_27a \ A_27d \ A_27c \ A_27b) \ V1abs1) \ V5rep2))))))))))
\end{aligned} \tag{8}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& \quad A_27a\ A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Equotient_2Erespects \\
& \quad (A_27b^{A_27a})\ 2)\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ A_27b) \\
& \quad V0R1)\ V3R2))\ V6f)) \wedge ((p\ (ap\ (ap\ (c_2Equotient_2Erespects\ (A_27b^{A_27a}) \\
& \quad 2)\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ A_27b)\ V0R1)\ V3R2)) \\
& \quad V7g)) \wedge ((ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c\ A_27b\ A_27a \\
& \quad A_27d)\ V2rep1)\ V4abs2)\ V6f) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ V7g))))))))))
\end{aligned}$$