# thm_2Equotient_2EFUN__REL__IMP (TMJvdcybCdi4f4WvFL6nJg8UEo5dW6pNJxq)

October 26, 2020

**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

**Definition 4** *We define* c_2Equotient_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

**Definition 5** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 6** *We define* c_2Equotient_2E_3D_3D_3D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27a}$

**Definition 7** *We define* c_2Ecombin_2EW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

**Definition 8** *We define* c_2Equotient_2Erespects *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\mathsf{c\_2Ecombin\_2EW}\ A\_27a\ A\_27b$

**Definition 9** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

**Definition 10** *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ($$
$$(2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in$$
$$(A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)$$
$$V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in$$
$$(A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}$$
$$A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}).$$
$$(\forall V7g \in (A\_27b^{A\_27a}).(((p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}$$
$$(A\_27b^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)$$
$$V0R1)\ V3R2))\ V6f)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (A\_27b^{A\_27a})$$
$$2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V3R2))$$
$$V7g))) \Rightarrow (((ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27b\ A\_27a$$
$$A\_27d)\ V2rep1)\ V4abs2)\ V6f) = (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}$$
$$A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ V7g)) \Leftrightarrow (\forall V8x \in A\_27a.$$
$$(\forall V9y \in A\_27a.((p\ (ap\ (ap\ V0R1\ V8x)\ V9y)) \Rightarrow (p\ (ap\ (ap\ V3R2\ (ap$$
$$V6f\ V8x))\ (ap\ V7g\ V9y)))))))))))))))))))) \tag{1}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ($$
$$(2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in$$
$$(A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)$$
$$V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in$$
$$(A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}$$
$$A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}).$$
$$(\forall V7g \in (A\_27b^{A\_27a}).(((p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}$$
$$(A\_27b^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)$$
$$V0R1)\ V3R2))\ V6f)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (A\_27b^{A\_27a})$$
$$2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V3R2))$$
$$V7g)) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27b\ A\_27a$$
$$A\_27d)\ V2rep1)\ V4abs2)\ V6f) = (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}$$
$$A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ V7g)))) \Rightarrow (\forall V8x \in$$
$$A\_27a.(\forall V9y \in A\_27a.((p\ (ap\ (ap\ V0R1\ V8x)\ V9y)) \Rightarrow (p\ (ap\ (ap$$
$$V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))))))))))))$$