

thm_2Equotient_2EIDENTITY__EQUIV (TM- RXzSAnPGt9LDWDPyHzvuJBeiCQxHdLLCZ)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 5 We define `c_2Equotient_2EEQUIV` to be $\lambda A. 27a : \iota. \lambda V 0E \in ((2^{A-27a})^{A-27a}). (\text{ap } (\text{c_2Ebool_2E_21 } (2^{A-27a})))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$(\forall V 0t1 \in 2. (\forall V 1t2 \in 2. (((p \Rightarrow V 0t1) \Rightarrow (p \Rightarrow V 1t2)) \Rightarrow (((p \Rightarrow V 1t2) \Rightarrow (p \Rightarrow V 0t1)) \Rightarrow ((p \Rightarrow V 0t1) \Leftrightarrow (p \Rightarrow V 1t2)))))) \tag{2}$$

Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow (\forall V 0x \in A. 27a. ((V 0x = V 0x) \Leftrightarrow \text{True})) \tag{3}$$

Theorem 1

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow (p \Rightarrow (\text{ap } (\text{c_2Equotient_2EEQUIV } A. 27a) (\text{c_2Emin_2E_3D } A. 27a)))$$