

thm_2Equotient_2EIN_FUN (TMHiBxBxp- kkY3UmPoJxTiqGLqoRjwVzsVC2)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2EIN` to be $\lambda A_{.27a} : \iota. (\lambda V0x \in A_{.27a}. (\lambda V1f \in (2^{A_{.27a}}). (ap V1f V0x)))$

Definition 3 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_{.27a} : \iota. (\lambda V0P \in (2^{A_{.27a}}). (ap (ap (c_2Emin_2E_3D (2^{A_{.27a}})$

Definition 5 We define `c_2Equotient_2E_2D_2D_3E` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda A_{.27c} : \iota. \lambda A_{.27d} : \iota. \lambda V0f$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. ((V0x = V0x) \Leftrightarrow True)) \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. nonempty A_{.27a} \Rightarrow \forall A_{.27b}. nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\ & nonempty A_{.27c} \Rightarrow \forall A_{.27d}. nonempty A_{.27d} \Rightarrow (\forall V0f \in (A_{.27c}^{A_{.27a}}). \\ & (\forall V1g \in (A_{.27d}^{A_{.27b}}). (\forall V2h \in (A_{.27b}^{A_{.27c}}). (\forall V3x \in \\ & A_{.27a}. ((ap (ap (ap (ap (c_2Equotient_2E_2D_2D_3E A_{.27a} A_{.27b} A_{.27c} \\ & A_{.27d}) V0f) V1g) V2h) V3x) = (ap V1g (ap V2h (ap V0f V3x)))))))))) \tag{3} \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}. nonempty A_{.27a} \Rightarrow \forall A_{.27b}. nonempty A_{.27b} \Rightarrow (\\ & \forall V0f \in (A_{.27b}^{A_{.27a}}). (\forall V1g \in (2^2). (\forall V2s \in (\\ & 2^{A_{.27b}}). (\forall V3x \in A_{.27a}. ((p (ap (ap (c_2Ebool_2EIN A_{.27a}) \\ & V3x) (ap (ap (ap (c_2Equotient_2E_2D_2D_3E A_{.27a} 2 A_{.27b} 2) V0f) \\ & V1g) V2s))) \Leftrightarrow (p (ap V1g (ap (ap (c_2Ebool_2EIN A_{.27b}) (ap V0f V3x)) \\ & V2s)))))))))) \end{aligned}$$