

# thm\_2Equotient\_2EI\_PRS (TMHM- fkH7yLQ8fEZ7dDpTqHkQnZHyYfQjK2h)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 4** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) P) V0P))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (V0t1 V1t2)))$

**Definition 9** We define  $c\_2Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27b}).\lambda V1abs \in (A\_27b^{A\_27a}).\lambda V2rep \in (A\_27a^{A\_27b}).((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT A\_27a A\_27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3a \in A\_27b.((ap V1abs (ap V2rep V3a)) = V3a))))))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap (c\_2Ecombin\_2EI A\_27a) V0x) = V0x)) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27b}).(\forall V1abs \in (A\_27b^{A\_27a}).(\forall V2rep \in (A\_27a^{A\_27b}).((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT A\_27a A\_27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3a \in A\_27b.((ap V1abs (ap V2rep V3a)) = V3a))))))) \tag{4}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs \in (A\_27b^{A\_27a}). \\ & \quad (\forall V2rep \in (A\_27a^{A\_27b}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3e \in A\_27b. ((ap\ (c\_2Ecombin\_2EI \\ & \quad A\_27b)\ V3e) = (ap\ V1abs\ (ap\ (c\_2Ecombin\_2EI\ A\_27a)\ (ap\ V2rep\ V3e)))))))))) \end{aligned}$$