

thm_2Equotient_2LEFT__RES__EXISTS__REGULAR (TMK5KXqNdAJwcczm6aXYWeXhzE7s4y8cNMe)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2EIN` to be $\lambda A_{27a} : \iota.(\lambda V0x \in A_{27a}.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 4 We define `c_2Ebool_2EET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (V0P) (V1x)) (V1x))))$

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A_{27a}) (V0P))))$

Definition 9 We define `c_2Ebool_2ERES__EXISTS` to be $\lambda A_{27a} : \iota.(\lambda V0p \in (2^{A-27a}).(\lambda V1m \in (2^{A-27a}).(ap (c_2Emin_2E_40 A_{27a}) (V0p))))$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in A_{27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (1)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1x \in A_{27a}.((p (ap (ap (c_2Ebool_2EIN A_{27a}) V1x) V0P)) \Leftrightarrow (p (ap V0P V1x)))))) \quad (2)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1f \in (2^{A-27a}).((p (ap (ap (c_2Ebool_2ERES__EXISTS A_{27a}) V0P) V1f)) \Leftrightarrow (\exists V2x \in A_{27a}.((p (ap (ap (c_2Ebool_2EIN A_{27a}) V2x) V0P)) \wedge (p (ap V1f V2x)))))))) \quad (3)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1R \in \\ & (2^{A_27a}). (\forall V2Q \in (2^{A_27a}). ((\forall V3x \in A_27a. ((p (\\ ap\ V1R\ V3x)) \Rightarrow ((p (ap\ V2Q\ V3x)) \Rightarrow (p (ap\ V0P\ V3x)))))) \Rightarrow ((p (ap (ap (c_2Ebool_2ERES_EXISTS \\ A_27a)\ V1R)\ V2Q)) \Rightarrow (p (ap (c_2Ebool_2E_3F\ A_27a)\ V0P)))))) \end{aligned}$$