thm_2Equotient_2ERESPECTS__MP (TMaNzttd6gLXe2veL8Yd66ZG6xSqyrCg1i3)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Equotient_2E_3D_3D_3D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27a}$

Definition 7 *We define* c_2Ecombin_2EW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

Definition 8 *We define* c_2Equotient_2Erespects *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\mathsf{c\_2Ecombin\_2EW}\ A\_27a\ A\_27b$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V2f \in (A\_27b^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (A\_27b^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V1R2))\ V2f)) \Leftrightarrow (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.((p\ (ap\ (ap\ V0R1\ V3x)\ V4y)) \Rightarrow (p\ (ap\ (ap\ V1R2\ (ap\ V2f\ V3x))\ (ap\ V2f\ V4y))))))))))) \tag{1}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V2f \in (A\_27b^{A\_27a}).(\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(((p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (A\_27b^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V1R2))\ V2f)) \wedge (p\ (ap\ (ap\ V0R1\ V3x)\ V4y))) \Rightarrow (p\ (ap\ (ap\ V1R2\ (ap\ V2f\ V3x))\ (ap\ V2f\ V4y)))))))))$$