

thm_2Equotient_2ERESPECTS_o (TM- MJYLi7M3Zj8BrPoRaEi2aCUpH5LobH8PT)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}).$

Definition 7 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27b}).$

Definition 8 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x \in (A_27a^{A_27b}).$

Definition 9 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b)$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\ & (\forall V2x \in A_27c.((ap (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) \\ & V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0R1 \in ((2^{A_27a})^{A_27a}).(\forall V1R2 \in ((2^{A_27b})^{A_27b}). \\ & (\forall V2f \in (A_27b^{A_27a}).((p (ap (ap (c_2Equotient_2Erespects \\ & (A_27b^{A_27a} 2) (ap (ap (c_2Equotient_2E_3D_3D_3D_3E A_27a A_27b) \\ & V0R1) V1R2)) V2f)) \Leftrightarrow (\forall V3x \in A_27a.(\forall V4y \in A_27a.((\\ & p (ap (ap V0R1 V3x) V4y)) \Rightarrow (p (ap (ap V1R2 (ap V2f V3x)) (ap V2f V4y)))))))))) \end{aligned} \quad (2)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0R1 \in ((2^{A_27a})^{A_27a}).(\forall V1R2 \in \\ & \quad ((2^{A_27b})^{A_27b}).(\forall V2R3 \in ((2^{A_27c})^{A_27c}).(\forall V3f \in \\ & (A_27c^{A_27b}).(\forall V4g \in (A_27b^{A_27a}).(((p\ (ap\ (ap\ (c_2Equotient_2Erespects \\ & (A_27c^{A_27b})\ 2)\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27b\ A_27c) \\ & V1R2)\ V2R3))\ V3f)) \wedge (p\ (ap\ (ap\ (c_2Equotient_2Erespects\ (A_27b^{A_27a}) \\ & 2)\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ A_27b)\ V0R1)\ V1R2)) \\ & V4g))) \Rightarrow (p\ (ap\ (ap\ (c_2Equotient_2Erespects\ (A_27c^{A_27a})\ 2)\ (\\ & ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ A_27c)\ V0R1)\ V2R3)) \\ & (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27c\ A_27b)\ V3f)\ V4g))))))))) \end{aligned}$$