

thm_2Equotient_2ERES_ABSTRACT_RSP
(TMQBMCf4BmYvWB6QetnaLKBT9XK8WmJATtT)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V0t \in 2.V0t)) (\lambda V1t \in 2.V1t)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 8 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1R \in ((2^{A-27b})^{A-27b}).inj_o$

Definition 9 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A-27a})^{A-27a}).\lambda V1R2 \in ((2^{A-27b})^{A-27b}).inj_o$

Definition 10 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A-27a})^{A-27a}).(\lambda V1x \in A_27a.\lambda V2x \in A_27b.inj_o (V0f (V1x V2x))))$

Definition 11 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b)$

Let $c_2Ebool_2ERES_ABSTRACT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Ebool_2ERES_ABSTRACT A_27a A_27b \in (((A_27b^{A-27a})^{A-27a})^{A-27a}) \quad (1)$$

Definition 12 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Assume the following.

$$True \quad (2)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p \ V0t)))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \forall V0f \in ((A_27b^{A_27a})^{A_27a}).(\forall V1x \in A_27a.((ap \ (ap \\ & (c_2Ecombin_2EW \ A_27a \ A_27b) \ V0f) \ V1x) = (ap \ (ap \ V0f \ V1x) \ V1x)))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1x \in \\ & A_27a.((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V1x) \ V0P)) \Leftrightarrow (p \ (ap \ V0P \ V1x)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\ & (\forall V2rep \in (A_27a^{A_27b}).((p \ (ap \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \\ & A_27a \ A_27b) \ V0R) \ V1abs) \ V2rep)) \Rightarrow (\forall V3r \in A_27a.(\forall V4s \in \\ & A_27a.((p \ (ap \ (ap \ V0R \ V3r) \ V4s)) \Leftrightarrow ((p \ (ap \ (ap \ V0R \ V3r) \ V3r)) \wedge ((p \ (ap \\ & (ap \ V0R \ V4s) \ V4s)) \wedge ((ap \ V1abs \ V3r) = (ap \ V1abs \ V4s)))))))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \forall V0p \in (2^{A_27a}).(\forall V1m \in (A_27b^{A_27a}).(\forall V2x \in \\ & A_27a.((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V2x) \ V0p)) \Rightarrow ((ap \ (ap \ (ap \\ & (c_2Ebool_2ERES_ABSTRACT \ A_27a \ A_27b) \ V0p) \ V1m) \ V2x) = (ap \ V1m \\ & V2x)))))) \end{aligned} \quad (8)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\ & \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\ & \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\ & \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\ & \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6f1 \in (A_27b^{A_27a}). \\ & \quad (\forall V7f2 \in (A_27b^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\ & \quad A_27a\ A_27b)\ V0R1)\ V3R2)\ V6f1)\ V7f2))) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\ & \quad A_27a\ A_27b)\ V0R1)\ V3R2)\ (ap\ (ap\ (c_2Ebool_2ERES_ABSTRACT\ A_27a \\ & \quad A_27b)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R1))\ V6f1))\ (ap \\ & \quad (ap\ (c_2Ebool_2ERES_ABSTRACT\ A_27a\ A_27b)\ (ap\ (c_2Equotient_2Erespects \\ & \quad A_27a\ 2)\ V0R1))\ V7f2))))))))))))) \end{aligned}$$