thm_2Equotient_2ERES__EXISTS__EQUIV (TMXq6UZeWMhqEzuZhjNjaJcwRXkRtAR2L1R)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 5 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 6 *We define* c_2Ebool_2ERES__FORALL *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).(a$

Definition 7 *We define* c_2Ecombin_2EW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

Definition 8 *We define* c_2Equotient_2Erespects *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(c\_2Ecombin\_2EW\ A\_27a\ A\_27b$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

Definition 12 *We define* c_2Ebool_2ERES__EXISTS *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).($

Definition 13 *We define* c_2Equotient_2ERES__EXISTS__EQUIV *to be* $\lambda A\_27a : \iota.(\lambda V0R \in ((2^{A\_27a})^{A\_27a}).$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1m \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Equotient\_2ERES\_\_EXISTS\_\_EQUIV\ A\_27a)\ V0R)\ V1m)) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_\_EXISTS\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ (\lambda V2x \in A\_27a.(ap\ V1m\ V2x)))) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ (\lambda V3x \in A\_27a.(ap\ (ap\ (c\_2Ebool\_2ERES\_\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ (\lambda V4y \in A\_27a.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ V1m\ V3x))\ (ap\ V1m\ V4y)))\ (ap\ (ap\ V0R\ V3x)\ V4y)))))))))))$$