thm_2Equotient_2ERES__EXISTS__UNIQUE__REGULAR__SAME (TMThHB8Pk5gDH68NEcQHw1NDFmf9LM3cSDu)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Equotient_2E_3D_3D_3D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27a}$

Definition 8 *We define* c_2Ecombin_2EW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

Definition 9 *We define* c_2Equotient_2Erespects *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\mathsf{c\_2Ecombin\_2EW}\ A\_27a\ A\_27b$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 12 *We define* c_2Ebool_2ERES__FORALL *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).($

Definition 13 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 15 *We define* c_2Ebool_2ERES__EXISTS *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).($

Definition 16 *We define* c_2Equotient_2ERES__EXISTS__EQUIV *to be* $\lambda A\_27a : \iota.(\lambda V0R \in ((2^{A\_27a})^{A\_27a}).$

Definition 17 *We define* c_2Ebool_2ERES__EXISTS__UNIQUE *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0P)) \Leftrightarrow (p\ (ap\ V0P\ V1x))))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R)\ V1x)) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V1x)\ V1x))))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1m \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2ERES\_\_EXISTS\_\_EQUIV}\ A\_27a)\ V0R)\ V1m)) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_EXISTS}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (\lambda V2x \in A\_27a.(ap\ V1m\ V2x)))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (\lambda V3x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (\lambda V4y \in A\_27a.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ V1m\ V3x))\ (ap\ V1m\ V4y)))\ (ap\ (ap\ V0R\ V3x)\ V4y)))))))))))) \tag{6}$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1P\_27 \in 2.(\forall V2Q \in 2.(\forall V3Q\_27 \in 2.((((p\ V0P) \Rightarrow (p\ V2Q)) \wedge ((p\ V1P\_27) \Rightarrow (p\ V3Q\_27))) \Rightarrow (((p\ V0P) \wedge (p\ V1P\_27)) \Rightarrow ((p\ V2Q) \wedge (p\ V3Q\_27)))))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1f \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V0P)\ V1f)) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0P)) \Rightarrow (p\ (ap\ V1f\ V2x))))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1f \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_EXISTS}\ A\_27a)\ V0P)\ V1f)) \Leftrightarrow (\exists V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0P)) \wedge (p\ (ap\ V1f\ V2x))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1f \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_EXISTS\_\_UNIQUE}\ A\_27a)\ V0P)\ V1f)) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_EXISTS}\ A\_27a)\ V0P)\ (\lambda V2x \in A\_27a.(ap\ V1f\ V2x)))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V0P)\ (\lambda V3x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V0P)\ (\lambda V4y \in A\_27a.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ V1f\ V3x))\ (ap\ V1f\ V4y)))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V3x)\ V4y))))))))))) \tag{10}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))\ V1P)\ V2Q)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_EXISTS\_\_UNIQUE}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2ERES\_\_EXISTS\_\_EQUIV}\ A\_27a)\ V0R)\ V2Q)))))))$$