

thm_2Equotient__list_2EAPPEND__RSP
(TMQjq3RL4jQHMy4NAX3UFy5hRrfrZ36Dr4q)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1$

Definition 7 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2Eappend : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2Eappend A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $c_2Elist_2Econs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2Econs A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2Eenil : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2Eenil A_27a \in (ty_2Elist_2Elist A_27a) \quad (4)$$

Let $c_2Elist_2ELIST_REL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2ELIST_REL\ A_27a\ A_27b \in (((2^{(ty_2Elist_2Elist\ A_27b)})^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27b})^{A_27a}}) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\ & A_27a).(ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) \\ & V0l) = V0l) \wedge (\forall V1l \in (ty_2Elist_2Elist\ A_27a).(\forall V2l2 \in \\ & (ty_2Elist_2Elist\ A_27a).(\forall V3h \in A_27a.((ap\ (ap\ (c_2Elist_2EAPPEND \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ V1l))\ V2l2) = (ap\ (ap \\ & (c_2Elist_2ECONS\ A_27a)\ V3h)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a) \\ & V1l)\ V2l2)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a).(p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A.27b})^{A.27a}). (\forall V1a \in A.27a. (\forall V2as \in \\
& \quad (ty_2Elist_2Elist\ A.27a). (\forall V3b \in A.27b. (\forall V4bs \in \\
& \quad (ty_2Elist_2Elist\ A.27b). (((p\ (ap\ (ap\ (ap\ (c.2Elist_2ELIST_REL \\
& \quad A.27a\ A.27b)\ V0R)\ (c.2Elist_2ENIL\ A.27a))\ (c.2Elist_2ENIL\ A.27b))) \Leftrightarrow \\
& \quad True) \wedge (((p\ (ap\ (ap\ (ap\ (c.2Elist_2ELIST_REL\ A.27a\ A.27b)\ V0R) \\
& \quad (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V1a)\ V2as))\ (c.2Elist_2ENIL\ A.27b))) \Leftrightarrow \\
& \quad False) \wedge (((p\ (ap\ (ap\ (ap\ (c.2Elist_2ELIST_REL\ A.27a\ A.27b)\ V0R) \\
& \quad (c.2Elist_2ENIL\ A.27a))\ (ap\ (ap\ (c.2Elist_2ECONS\ A.27b)\ V3b)\ V4bs))) \Leftrightarrow \\
& \quad False) \wedge ((p\ (ap\ (ap\ (ap\ (c.2Elist_2ELIST_REL\ A.27a\ A.27b)\ V0R) \\
& \quad (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V1a)\ V2as))\ (ap\ (ap\ (c.2Elist_2ECONS \\
& \quad A.27b)\ V3b)\ V4bs))) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V1a)\ V3b)) \wedge (p\ (ap\ (ap\ (ap\ (c.2Elist_2ELIST_REL \\
& \quad A.27a\ A.27b)\ V0R)\ V2as)\ V4bs))))))))) \\
& \hspace{15em} (12)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A.27a})^{A.27a}). (\forall V1abs \in (A.27b^{A.27a}). \\
& \quad (\forall V2rep \in (A.27a^{A.27b}). ((p\ (ap\ (ap\ (ap\ (c.2Equotient_2EQUOTIENT \\
& \quad A.27a\ A.27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3l1 \in (ty_2Elist_2Elist \\
& \quad A.27a). (\forall V4l2 \in (ty_2Elist_2Elist\ A.27a). (\forall V5m1 \in \\
& \quad (ty_2Elist_2Elist\ A.27a). (\forall V6m2 \in (ty_2Elist_2Elist\ A.27a). \\
& \quad (((p\ (ap\ (ap\ (ap\ (c.2Elist_2ELIST_REL\ A.27a\ A.27a)\ V0R)\ V3l1)\ V4l2)) \wedge \\
& \quad (p\ (ap\ (ap\ (ap\ (c.2Elist_2ELIST_REL\ A.27a\ A.27a)\ V0R)\ V5m1)\ V6m2))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (ap\ (c.2Elist_2ELIST_REL\ A.27a\ A.27a)\ V0R)\ (ap\ (ap\ (c.2Elist_2EAPPEND \\
& \quad A.27a)\ V3l1)\ V5m1))\ (ap\ (ap\ (c.2Elist_2EAPPEND\ A.27a)\ V4l2)\ V6m2))))))))) \\
\end{aligned}$$