

# thm\_2Equotient\_list\_2ECONS\_\_RSP (TMTA- maQpzTz7LkUmSTv3VUaYQM64ubH6vd3)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 3** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a})) (\lambda V1t \in 2. V1t)) (\lambda V2t \in 2. V2t)))$

**Definition 5** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 6** We define `c_2Equotient_2EQUOTIENT` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1R \in ((2^{A_27b})^{A_27b}). \text{inj\_o } (V0R = V1R)$

**Definition 7** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 8** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2E_2F } 2)) (\lambda V1t \in 2. V1t)))$

**Definition 9** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } (2^{A_27a})) (\lambda V1x \in 2^{A_27a}. V1x))))$

**Definition 11** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Let `ty_2Elist_2Elist` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty\_2Elist\_2Elist } A0) \quad (1)$$

Let `c_2Elist_2ECONS` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c\_2Elist\_2ECONS } A_27a \in (((\text{ty\_2Elist\_2Elist } A_27a)^{(\text{ty\_2Elist\_2Elist } A_27a)})^{A_27a}) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (3)$$

Let  $c\_2Elist\_2ELIST\_REL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b \in (((2^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27b})^{A\_27a}}) \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (6) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (7) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(V0l = (c\_2Elist\_2ENIL\ A\_27a)) \vee (\exists V1h \in A\_27a.( \\ & \exists V2t \in (ty\_2Elist\_2Elist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1h)\ V2t)))))) \quad (8) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0R \in ((2^{A\_27b})^{A\_27a}).(\forall V1a \in A\_27a.(\forall V2as \in \\ & (ty\_2Elist\_2Elist\ A\_27a).(\forall V3b \in A\_27b.(\forall V4bs \in \\ & (ty\_2Elist\_2Elist\ A\_27b).(((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27b))) \Leftrightarrow \\ & True) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (c\_2Elist\_2ENIL\ A\_27b))) \Leftrightarrow \\ & False) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3b)\ V4bs))) \Leftrightarrow \\ & False) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3b)\ V4bs))) \Leftrightarrow \\ & ((p\ (ap\ (ap\ V0R\ V1a)\ V3b)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R)\ V2as)\ V4bs)))))) \quad (9) \end{aligned}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs \in (A\_27b^{A\_27a}). \\ & (\forall V2rep \in (A\_27a^{A\_27b}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3t1 \in (ty\_2Elist\_2Elist \\ & \quad A\_27a). (\forall V4t2 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V5h1 \in \\ & \quad A\_27a. (\forall V6h2 \in A\_27a. (((p\ (ap\ (ap\ V0R\ V5h1)\ V6h2)) \wedge (p\ (ap \\ & \quad (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27a)\ V0R)\ V3t1)\ V4t2)))) \Rightarrow \\ & (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27a)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ & \quad A\_27a)\ V5h1)\ V3t1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V6h2)\ V4t2)))))))))) \end{aligned}$$