

thm\_2Equotient\_\_list\_2EFILTER\_\_RSP  
(TMbhHVz9gzfgjSyJbGajudLEfBXop76ouR6)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

**Definition 5** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

**Definition 6** We define `c_2Equotient_2EQUOTIENT` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27b}).\lambda V$

**Definition 7** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

**Definition 8** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 9** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

**Definition 10** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define `c_2Ebool_2ECOND` to be  $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let `ty_2Elist_2Elist` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let `c_2Elist_2EFILTER` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFILTER A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))^{(2^{A_27a})}) \quad (2)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (3)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (4)$$

Let  $c\_2Elist\_2ELIST\_REL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b \in (((2^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27b})^{A\_27a}}) \quad (5)$$

**Definition 12** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R1 \in ((2^{A\_27a})^{A\_27b})$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in \\
& A.27a.(((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF) \\
& V0t1) V1t2) = V1t2))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((ap ( \\
& ap (c.2Elist.2EFILTER A.27a) V0P) (c.2Elist.2ENIL A.27a)) = (c.2Elist.2ENIL \\
& A.27a))) \wedge (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in \\
& (ty.2Elist.2Elist A.27a).((ap (ap (c.2Elist.2EFILTER A.27a) \\
& V1P) (ap (ap (c.2Elist.2ECONS A.27a) V2h) V3t)) = (ap (ap (ap (c.2Ebool.2ECOND \\
& (ty.2Elist.2Elist A.27a) (ap V1P V2h)) (ap (ap (c.2Elist.2ECONS \\
& A.27a) V2h) (ap (ap (c.2Elist.2EFILTER A.27a) V1P) V3t)))) (ap (ap \\
& (c.2Elist.2EFILTER A.27a) V1P) V3t))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist A.27a)}). \\
& (((p (ap V0P (c.2Elist.2ENIL A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\
& A.27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A.27a.(p (ap V0P (ap (ap ( \\
& c.2Elist.2ECONS A.27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& A.27a).(p (ap V0P V3l))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27b})^{A\_27a}). (\forall V1a \in A\_27a. (\forall V2as \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27a). (\forall V3b \in A\_27b. (\forall V4bs \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27b). (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL \\
& \quad A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27b)))) \Leftrightarrow \\
& \quad True) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R) \\
& \quad (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (c\_2Elist\_2ENIL\ A\_27b)))) \Leftrightarrow \\
& \quad False) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R) \\
& \quad (c\_2Elist\_2ENIL\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3b)\ V4bs)))) \Leftrightarrow \\
& \quad False) \wedge ((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R) \\
& \quad (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& \quad A\_27b)\ V3b)\ V4bs)))) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V1a)\ V3b)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL \\
& \quad A\_27a\ A\_27b)\ V0R)\ V2as)\ V4bs))))))))) \\
& \hspace{15em} (18)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3P1 \in (2^{A\_27a}). (\forall V4P2 \in \\
& \quad (2^{A\_27a}). (\forall V5l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V6l2 \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27a). (((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E \\
& \quad A\_27a\ 2)\ V0R)\ (c\_2Emin\_2E\_3D\ 2))\ V3P1)\ V4P2)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL \\
& \quad A\_27a\ A\_27a)\ V0R)\ V5l1)\ V6l2)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL \\
& \quad A\_27a\ A\_27a)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ V3P1)\ V5l1)) \\
& \quad (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ V4P2)\ V6l2))))))))) \\
\end{aligned}$$