

# thm\_2Equotient\_list\_2EFOLDL\_PRS (TMZqoVRjCfsMDavuzJEwe3pwqS42Sn61zww)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EMAP A\_27a A\_27b \in (((ty\_2Elist\_2Elist A\_27b)^{(ty\_2Elist\_2Elist A\_27a)})^{(A\_27b^{A\_27a})}) \quad (2)$$

Let  $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EFOLDL A\_27a A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \quad (3)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (4)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (5)$$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in 2.$

**Definition 6** We define  $c\_2Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V$

**Definition 7** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{8}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( & \\ (\forall V0f \in (A\_27b^{A\_27a}).((ap (ap (c\_2Elist\_2EMAP A\_27a A\_27b) & \\ V0f) (c\_2Elist\_2ENIL A\_27a)) = (c\_2Elist\_2ENIL A\_27b))) \wedge (\forall V1f \in & \\ (A\_27b^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist & \\ A\_27a).((ap (ap (c\_2Elist\_2EMAP A\_27a A\_27b) V1f) (ap (ap (c\_2Elist\_2ECONS & \\ A\_27a) V2h) V3t)) = (ap (ap (c\_2Elist\_2ECONS A\_27b) (ap V1f V2h)) & \\ (ap (ap (c\_2Elist\_2EMAP A\_27a A\_27b) V1f) V3t))))))))) & \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( & \\ (\forall V0f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V1e \in A\_27b.((ap ( & \\ ap (ap (c\_2Elist\_2EFOLDL A\_27a A\_27b) V0f) V1e) (c\_2Elist\_2ENIL & \\ A\_27a)) = V1e))) \wedge (\forall V2f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V3e \in & \\ A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist A\_27a). & \\ ((ap (ap (ap (c\_2Elist\_2EFOLDL A\_27a A\_27b) V2f) V3e) (ap (ap (c\_2Elist\_2ECONS & \\ A\_27a) V4x) V5l)) = (ap (ap (ap (c\_2Elist\_2EFOLDL A\_27a A\_27b) V2f) & \\ (ap (ap V2f V3e) V4x)) V5l))))))))) & \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist A\_27a)}). & \\ (((p (ap V0P (c\_2Elist\_2ENIL A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist & \\ A\_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A\_27a.(p (ap V0P (ap (ap ( & \\ c\_2Elist\_2ECONS A\_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist & \\ A\_27a).(p (ap V0P V3l)))))) & \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3a \in A\_27b.((ap\ V1abs \\
& \quad (ap\ V2rep\ V3a)) = V3a))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27c^{A\_27a}). \\
& \quad (\forall V1g \in (A\_27d^{A\_27b}).(\forall V2h \in (A\_27b^{A\_27c}).(\forall V3x \in \\
& A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ A\_27b\ A\_27c \\
& A\_27d)\ V0f)\ V1g)\ V2h)\ V3x) = (ap\ V1g\ (ap\ V2h\ (ap\ V0f\ V3x)))))))))
\end{aligned} \tag{13}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6l \in (ty\_2Elist\_2Elist \\
& \quad A\_27d).(\forall V7f \in ((A\_27c^{A\_27d})^{A\_27c}).(\forall V8e \in A\_27c. \\
& ((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27d\ A\_27c)\ V7f)\ V8e)\ V6l) = (ap\ V1abs1 \\
& (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\
& A\_27a\ (A\_27c^{A\_27d})\ A\_27c\ (A\_27a^{A\_27b}))\ V1abs1)\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\
& A\_27b\ A\_27c\ A\_27d\ A\_27a)\ V4abs2)\ V2rep1))\ V7f))\ (ap\ V2rep1\ V8e)) \\
& (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27d\ A\_27b)\ V5rep2)\ V6l))))))))))
\end{aligned}$$