thm_2Equotient__list_2EFOLDL__RSP (TMXp1eg5YtvAfVy1mT5qbuczzn1GK4netbx)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \tag{2}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{3}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{4}$$

Let $c\_2Elist\_2ELIST\_\_REL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b \in (((2^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((2^{A\_27b})^{A\_27a})}) \tag{5}$$

Definition 9 *We define* c_2Equotient_2E_3D_3D_3D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27a}$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V1e \in A\_27b.((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (c\_2Elist\_2ENIL\ A\_27a)) = V1e))) \wedge (\forall V2f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V3e \in A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V2f)\ (ap\ (ap\ V2f\ V3e)\ V4x))\ V5l))))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).\ (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l)))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27b})^{A\_27a}).(\forall V1a \in A\_27a.(\forall V2as \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3b \in A\_27b.(\forall V4bs \in (ty\_2Elist\_2Elist\ A\_27b).(((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27b))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (c\_2Elist\_2ENIL\ A\_27b))) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3b)\ V4bs))) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3b)\ V4bs))) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V1a)\ V3b)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ V2as)\ V4bs)))))))))))) \tag{14}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6l1 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V7l2 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V8f1 \in ((A\_27a^{A\_27b})^{A\_27a}).(\forall V9f2 \in ((A\_27a^{A\_27b})^{A\_27a}).(\forall V10e1 \in A\_27a.(\forall V11e2 \in A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ (A\_27a^{A\_27b}))\ V0R1)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27b\ A\_27a)\ V3R2)\ V0R1))\ V8f1)\ V9f2)) \wedge ((p\ (ap\ (ap\ V0R1\ V10e1)\ V11e2)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27b\ A\_27b)\ V3R2)\ V6l1)\ V7l2)))) \Rightarrow (p\ (ap\ (ap\ V0R1\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ V8f1)\ V10e1)\ V6l1))\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ V9f2)\ V11e2)\ V7l2))))))))))))))))))$$