thm_2Equotient__list_2ELIST__REL__REFL (TMJciKnBmCkuKc2fgdZPUiCAsCZRo263pFZ)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{2}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{3}$$

Let $c\_2Elist\_2ELIST\_\_REL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b \in (((2^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((2^{A\_27b})^{A\_27a})}) \tag{4}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27b})^{A\_27a}).(\forall V1a \in A\_27a.(\forall V2as \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3b \in A\_27b.(\forall V4bs \in (ty\_2Elist\_2Elist\ A\_27b).(((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27b))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (c\_2Elist\_2ENIL\ A\_27b))) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3b)\ V4bs))) \Leftrightarrow False) \wedge ((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3b)\ V4bs))) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V1a)\ V3b)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27b)\ V0R)\ V2as)\ V4bs)))))))))))) \tag{10}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Leftrightarrow ((ap\ V0R\ V1x) = (ap\ V0R\ V2y))))) \Rightarrow (\forall V3x \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_\_REL\ A\_27a\ A\_27a)\ V0R)\ V3x)\ V3x)))))$$