

thm_2Equotient_list_2EMAP__RSP (TMVvtKzsT3Yna7dmSFt6wsGhkFtMwqJTGHU)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EMAP A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)})^{(A_27b^{A_27a})}) \quad (2)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (4)$$

Let $c_2Elist_2ELIST_REL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2ELIST_REL A_27a A_27b \in (((2^{(ty_2Elist_2Elist A_27b)})^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27b})^{A_27a}}) \quad (5)$$

Definition 7 We define `c_2Equotient_2E_3D_3D_3D_3E` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27a})$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 9 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p V0t)))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}).((ap (ap (c_2Elist_2EMAP A_27a A_27b) \\ & V0f) (c_2Elist_2ENIL A_27a)) = (c_2Elist_2ENIL A_27b))) \wedge (\forall V1f \in \\ & (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\ & A_27a).((ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) (ap (ap (c_2Elist_2ECONS \\ & A_27a) V2h) V3t)) = (ap (ap (c_2Elist_2ECONS A_27b) (ap V1f V2h)) \\ & (ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) V3t)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist A_27a)}). \\ & (((p (ap V0P (c_2Elist_2ENIL A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_27a.(p (ap V0P (ap (ap (\\ & c_2Elist_2ECONS A_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a).(p (ap V0P V3l)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27b})^{A_27a}). (\forall V1a \in A_27a. (\forall V2as \in \\
& \quad (ty_2Elist_2Elist\ A_27a). (\forall V3b \in A_27b. (\forall V4bs \in \\
& \quad (ty_2Elist_2Elist\ A_27b). (((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL \\
& \quad A_27a\ A_27b)\ V0R)\ (c_2Elist_2ENIL\ A_27a))\ (c_2Elist_2ENIL\ A_27b)))) \Leftrightarrow \\
& \quad True) \wedge (((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27b)\ V0R) \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1a)\ V2as))\ (c_2Elist_2ENIL\ A_27b)))) \Leftrightarrow \\
& \quad False) \wedge (((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27b)\ V0R) \\
& \quad (c_2Elist_2ENIL\ A_27a))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V3b)\ V4bs)))) \Leftrightarrow \\
& \quad False) \wedge (((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27b)\ V0R) \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1a)\ V2as))\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27b)\ V3b)\ V4bs)))) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V1a)\ V3b)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL \\
& \quad A_27a\ A_27b)\ V0R)\ V2as)\ V4bs))))))))) \\
& \hspace{15em} (12)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}). (\forall V1abs1 \in (A_27c)^{A_27a}). (\forall V2rep1 \in \\
& \quad (A_27a)^{A_27c}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in \\
& \quad (A_27d)^{A_27b}). (\forall V5rep2 \in (A_27b)^{A_27d}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b)^{A_27a}). \\
& \quad (\forall V7g \in (A_27b)^{A_27a}). (\forall V8x \in A_27a. (\forall V9y \in \\
& \quad A_27a. (((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y)))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
& \hspace{15em} (13)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}). (\forall V1abs1 \in (A_27c)^{A_27a}). (\forall V2rep1 \in \\
& \quad (A_27a)^{A_27c}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in \\
& \quad (A_27d)^{A_27b}). (\forall V5rep2 \in (A_27b)^{A_27d}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6l1 \in (ty_2Elist_2Elist \\
& \quad A_27a). (\forall V7l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V8f1 \in \\
& \quad (A_27b)^{A_27a}). (\forall V9f2 \in (A_27b)^{A_27a}). (((p\ (ap\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ A_27b)\ V0R1)\ V3R2)\ V8f1)\ V9f2)) \wedge \\
& \quad (p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27a)\ V0R1)\ V6l1)\ V7l2)))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27b\ A_27b)\ V3R2)\ (ap\ (ap\ (\\
& \quad c_2Elist_2EMAP\ A_27a\ A_27b)\ V8f1)\ V6l1))\ (ap\ (ap\ (c_2Elist_2EMAP \\
& \quad A_27a\ A_27b)\ V9f2)\ V7l2))))))))) \\
\end{aligned}$$